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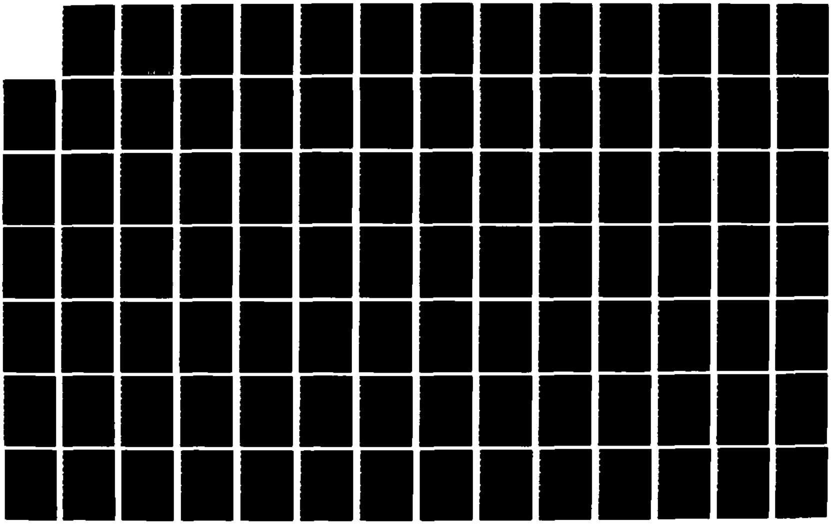

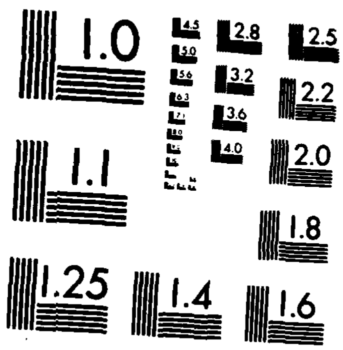

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THESIS ABSTRACT

THE OHIO STATE UNIVERSITY
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DEPARTMENT: Industrial and Systems Engineering DEGREE: M.S.

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TITLE: The Effectiveness of Jackson Networks as Control Variates for Queueing Network Simulation

Control variates based on Jackson networks are investigated for variance reduction in open queueing network simulations. Two types of control variates are studied: an external obtained from simulating a similar Jackson network, and a new analytic control derived from the parametric equations of the Jackson model. The analytic control eliminates the need for a second simulation required of true external controls. The analytic controls showed considerable promise for reducing the variance of server utilizations, but indicated the need for additional study of the effects of batch size and network structure on control variate performance. (98 pages)

Barry L. Nelson

BARRY L. NELSON, Ph.D
Adviser

THE EFFECTIVENESS OF JACKSON NETWORKS AS CONTROL VARIATES FOR QUEUEING NETWORK SIMULATION

A Thesis

Presented in Partial Fulfillment of the Requirements for the Degree Master of Science in the Graduate School of The Ohio State University

By

Anthony P. Sharon, B.S.

* * * * *

The Ohio State University

1986

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DEDICATION

To my wife and children

ACKNOWLEDGEMENTS

I wish to thank my adviser, Dr. Barry L. Nelson, his direction and counsel were the cornerstones of this research. I also wish to acknowledge Dr. John B. Neuhardt and Dr. Gordon M. Clark for their suggestions and recommendations. Thanks are also due to Wei-Ning Yang and Po-Yen Wu for their helpful discussions during the course of this study.

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CHAPTER I

INTRODUCTION

Basic queueing theory begins with an arrival process, a service mechanism, and a queue discipline. Practical applications can extend this beginning into a network where the nodes would be service mechanisms of one or more servers. Such applications include communication systems, computer time sharing processes, medical care facilities, assembly operations, and so on. The analysis of such networks involves the solution of large scale systems of equations and computational problems of large dimensions. Due to the intractability of the mathematical models, computer simulation is a commonly employed analysis approach.

Simulation, however, is an experimental approach rather than an analytical one, and presents a host of issues inherent in sampling. These issues include the choice of input distributions, statistical methods for analyzing output, the comparison of alternative systems, model verification and validation, and techniques used to improve the precision of estimators. The last issue is commonly referred to as variance reduction and is the topic of this research.

A simulation of queueing networks is partially driven by sampling realizations of random variables; therefore, the outputs produced are also random variables. These outputs are generally mapped into estimates of interest through an output function (e.g. a sample mean for instance). These estimators possess sampling distributions usually having unknown means. The precision of these estimators is measured by their variances: the smaller the variance the greater the precision. Therefore, reducing the variance is a method for increasing the efficiency of the simulation.

One technique employed for variance reduction is the use of control variates. This technique uses the correlation between specified random variables to achieve a variance reduction. One type of control variate is the external control, which is obtained by simulating a similar system whose performance measures can be analytically computed or closely approximated. A variance reduction can be obtained if the output of the second simulation is positively correlated with its counterpart from the original simulation.

A number of queueing networks can be categorized as Jackson networks, for which the analytical computation of various performance measures is possible. Jackson networks have been considered as possible control variates for simulating more general networks. There are at least two ways in which Jackson networks can be used to obtain control

variates. The first method, as described above, involves running a second simulation of a similar Jackson network and using the corresponding output of this second simulation as the control variate. This method is commonly referred to as external control variates. A formidable drawback with this approach is the cost of the second simulation.

The other method for obtaining a control variate is to use the difference between two performance measures calculated from the Jackson model as the control variate. One measure is computed by substituting the known input parameters into the Jackson equations. These parameters could be the mean arrival or service rates used to drive the simulation. The other performance measure is computed by substituting estimates of these same parameters obtained from the simulation into the Jackson equations. This type of control variate is referred to as an analytic control since it is obtained from an analytical operation rather than a second simulation. The advantage of this approach is eliminating the cost of the second simulation. This is a new approach.

The purpose of this research is to study the effectiveness of Jackson networks as external and analytic controls for queueing network simulations. The approach taken is to experiment with a small but representative set of networks

with an eye toward drawing general conclusions about the performance of this variance reduction technique. Nelson [15,16] notes that prior knowledge of the system in question is a key component in the selection of an appropriate variance reduction technique. The conclusions drawn from this research should provide the analyst some prior knowledge for selecting the appropriate variance reduction technique.

This research will attempt to add to this prior knowledge by studying the performance of Jackson based external and analytic controls on various queueing performance measures, by investigating the impact of the service distributions, traffic intenstity, and network structure. In addition, the suitability of automating this approach and areas of future research will be discussed. The remainder of this work includes a background on queueing networks and control variates, the methodology used in this research, results and conclusions.

CHAPTER II

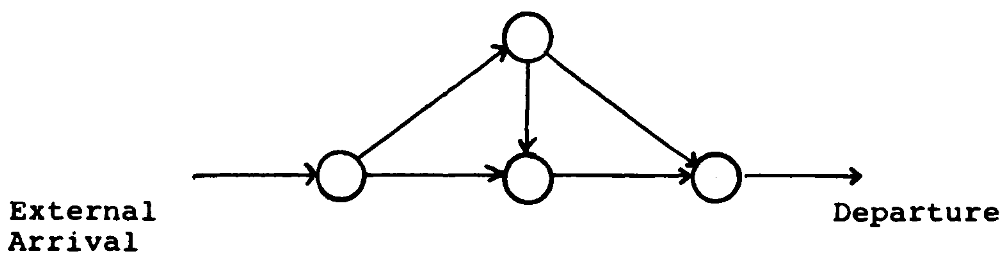
BACKGROUND

The purpose of this chapter is to present an integrated review of the literature relevant to this research. The review is divided into three sections: the first presents a brief introduction to queueing networks and formally defines a Jackson network; the second presents the theory and development of control variates, and the third discusses the results of the control variate techniques applied to queueing network simulations.

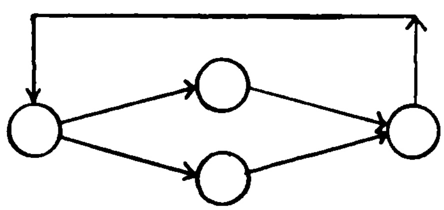
QUEUEING NETWORKS

In general, queueing networks are classified as open or closed networks. In an open network customers arrive from outside the network; this characteristic is called exogenous arrivals. In general customers may enter the network at any node. Customers then proceed through the network according to their needs or in some random manner and may depart the network from any node. A closed network is similar in structure to an open network; however, there are no exogenous arrivals and customers never depart the network.

There is always some fixed number of customers present in a closed network. Figure 1 shows examples of the open and closed network types. This network contains a number of points where customer routing decisions must be made. These points are called switches and their operation is governed by switch rules. These rules may be imposed externally to the system (e.g. a routing or dispatch form), internal to the system (e.g. the server at node 1 may determine whether a customer goes to node 2 or 3), or the rules may be determined by the customers (e.g. customer selects the shortest waiting line).



Open Network



Closed Network(Number of Customers= C)

<u>Figure 1</u>: Open and Closed Queueing Networks

While the systems in Figure 1 illustrate the idea of open and closed networks, more detailed symbols are needed to model more complicated structures. Consider the network in Figure 2.

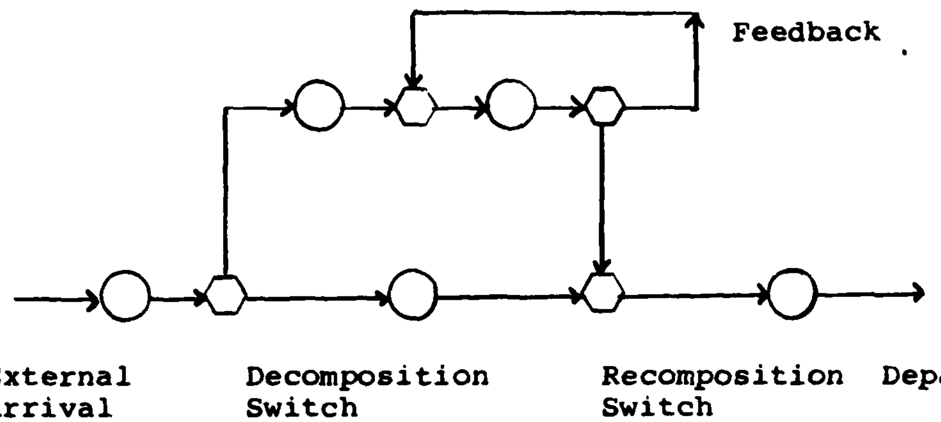


Figure 2: *Sample Queueing Network*

There are two basic types of switches: decomposition and recomposition. A decompostion switch splits a single stream of customers into a number of streams. A recomposition switch merges a number of streams into one superposed stream.

Another possible feature is feedback. A feedback point is one where customers may be directed to repeat a service node; direction is provided through feedback rules.

There are three principal methods for analyzing queueing networks: first, analyze the network as a whole; second, decompose the network into subnetworks; and third,

use computer simulation. In his survey paper Disney [3] notes that the difficulties in mathematically analyzing queueing networks arise from flow properties rather than physical properties. Once customers enter a network the combinatorial effect of service mechanisms, switch and feedback rules, and queue disciplines alter the flow within the system for the individual customer.

Many of the techniques for analyzing networks as a whole are based on the research of J.R. Jackson [7,8]. The major thrust in this area has been studying the queue length process and most of the known results are for steady-state behavior. The primary obstacles encountered are finding the solutions of large scale systems of equations.

The second approach, decomposition, attempts to break the network down into subnetworks whose characteristics are well known. The most commonly used point for decomposition is at the switches. There are two basic technical problems with this approach: first, determining the effect of switching rules on the stochastic properties of network flow; and second, determining the result of recombination of the subnetworks. The primary obstacles encountered are probabilistic as opposed to algebraic, and involve computational problems of large dimension.

The third approach, computer simulation is probably the most commonly employed for general networks. Simulation is

an experimental approach rather than an analytical approach. The obstacle faced is how to analyze the measures obtained from the simulation. Often simulation output is used to estimate a population mean. In general, the output is correlated and highly variable. Estimation and the control of the variance of estimators is important, and is reflected in the validity and width, respectively, of interval estimators of these popluation means.

The characteristics of a queueing network are principally determined by the arrival processes, service mechanisms, queue disciplines, switches, and feedback rules. The model formulated by Jackson [7] properly defines these characteristics so as to facilitate a generalization of the M/M/s queue (Kendall notation meaning exponential interarrival times/ exponential service times/s servers) to an interconnected open network of service nodes. The defining characteristics of a Jackson network are listed below:

1. The network contains more than one service node.
2. Each node can be a single or multiple server queue with each server having an identical exponential service time distribution. Service times are independent.
3. Arrivals from outside the network occur in a Poisson fashion. Outside arrivals to any node are independent.
4. Arrivals at any given node may come from outside the network or from any other nodes.

5. The effective arrival rate at every node is less than its potential service rate.
6. When a customer completes service at a node, he may leave the network or be routed to another node.
7. There is unlimited waiting space at every service node.
8. The queue discipline is first come first served.

Although the parameters are fairly restrictive, the model is still quite general. Subsets of Jackson networks are

1. A finite number of M/M/s queues in tandem; tandem networks have only one exogenous arrival point and one path through the network.
2. An acyclic network of M/M/s queues; these are networks where customers may visit a node only once.
3. A network of M/M/s queues with feedback.

Jackson proved the important result that in steady state conditions, each node in the network functions as an independent M/M/s queue with Poisson input. This fact allows the decomposition into subnetworks and the pursuant application of M/M/s results. Another important fact is that although feedback destroys the Poisson property of the input stream, the nodes of the Jackson network continue to function as though the input process was Poisson.

Lemoine [13] presented a survey of equilibrium results for general Jackson networks. If the network is open, the

equilibrium rate of flow through node i, e_i , is the sum of the external input rate, λ_i, and the total rate of internal inputs to node i. This balance equation can be written as

$$e_i = \lambda_i + \sum_{j=1}^{N} r_{ji}\, e_j \quad i=1,\ldots,N \tag{1}$$

where r_{ji} is the probability a customer is routed from node j to node i, and N is the number of service nodes in the network.

Since the effective arrival rate at each node i must be less than its potential service rate, or else the number of customers in the system will continually grow as time goes on, the traffic intensity ρ_i must satisfy

$$\rho_i = e_i/(s_i u_i) < 1 \qquad i=1,\ldots,N \tag{2}$$

where s_i is the number of servers and u_i is the service rate at node i.

Another equilibrium flow condition derived from the open network is that the total input flow rate must equal the external departure rate. For any node i, the probability that any customer leaves the network is

$$q_i = 1 - \sum_{j=1}^{N} r_{ij} \tag{3}$$

therefore,

$$\sum_{i=1}^{N} \lambda_i = \sum_{i=1}^{N} e_i \ q_i \tag{4}$$

In his work Jackson [7] used as a state variable a vector whose components represent the number of customers present at each node in the network. His analysis showed that under equilibrium conditions the probabilty of the system being in state k, p(k), could be factored into a product of the marginal probabilities:

$$p(k_1,\ldots,k_N)=p_1(k_1) \ \ldots \ p_N(k_N) \tag{5}$$

where

$$p_i(\emptyset)=\left[\sum_{k=\emptyset}^{s_i-1} \frac{(e_i/u_i)^k}{k!} + \frac{(e_i/u_i)^{s_i}}{s_i! \ (1-\rho_i)}\right]^{-1} \tag{6}$$

and

$$p_i(k)=\begin{cases} \dfrac{p_i(\emptyset) \ (e_i/u_i)^k}{k!} & k=\emptyset,1,\ldots,s_i \\[2ex] \dfrac{p_i(\emptyset) \ (e_i/u_i)^k}{s_i! \ (s_i)^{k-s_i}} & k>s_i \end{cases} \tag{7}$$

The preceding two equations can be recognized as those of the basic M/M/s model with effective arrival rate, e_i, replacing λ_i. This generalization permits the decomposition of the Jackson network into a collection of multi-server subnetworks.

Using the above results and Little's [14] formula the following measures of performance can be obtained:

Long run queue length $(LQ_i) = \dfrac{p_i(0)\,(e_i/u_i)^{s_i}\,(\rho_i)}{s_i!\,(1-\rho_i)^2}$ (8)

Long run node length $(L_i) = LQ_i + e_i/u_i$ (9)

The long run node length is the sum of the number of customers in service and the number in the queue.

Long run queue time $(WQ_i) = LQ_i/e_i$ (10)

Long run node time $(W_i) = WQ_i + 1/u_i$ (11)

Nelson [17] extended the results for Jackson networks by deriving the probability distribution for the total waiting time (excluding service time) for a customer to pass completely through the network. This result was obtained by the convolution of waiting time distributions at each node.

The sojourn time of a customer in a network is the time spent at each of the nodes visited (queue and service time combined) plus travel time between the nodes. Travel time in a Jackson network is assumed to be zero. For most networks the sojourn time problem is unsolved. Burke [1] and Reich [18] present results for small special case networks.

Gordon and Newell [6] analyze a closed network of N interconnected nodes and C customers. Each node has s_i, $i=1,\ldots,N$, parallel servers each with service rate u_i. The routing from node to node is the same as a Jackson network except customers do not depart the network. The system is equivalent to some open networks where the number of customers cannot exceed C. The authors' principal result was an expression for the equilibrium distribution at each node. The expression is factored into product terms for each node with the exception of an unknown normalizing constant that reflects the interaction between nodes.

Buzen [2] developed an iterative technique for determining the normalizing constant. He also derived the marginal distributions of the number of customers present at the nodes, the expected number of customers at each node and the steady state utilizations.

Solberg [19] developed a computationally efficient method for computing the normalizing constant.

In summary, the analysis of Jackson networks have the following limitations:

1. Service time distributions must be exponential.
2. Service nodes must have identical servers.
3. Only probabilistic routing between nodes is permitted.
4. Customer oriented performance measures such as sojourn times are difficult to obtain for other than special cases.
5. Travel time between nodes is assumed to be zero.

The scarcity of analytical results for other than special cases makes using network models difficult for practical applications. In general computer simulation often becomes the analysis approach and, as mentioned previously, estimators of the performance measures will possess some degree of variability. Reducing this variability to increase the estimator's precision becomes a major concern. One way of addressing this concern is the use of control variates.

CONTROL VARIATES

The central idea of control variates is to use the correlation between specified random variables to achieve a variance reduction. A random variable, C, is a control variable for the random variable Y, if it has a known expectation, γ, and is correlated with Y.

Let Y be an unbiased estimator of Θ, the quantity of interest, obtained from a single simulation run. Then for any constant b, an estimator of Y can be written as

$$Y(b) = Y - b(C - \gamma) \tag{12}$$

Equation (12) is also an unbiased estimator of O. The variance of Y(b) is given by

$$Var[Y(b)]=Var[Y] + b^2 Var[C] - 2bCov[Y,C] \tag{13}$$

which is the same as

$$Var[Y(b)]=Var[Y] + b^2 Var[C] - 2b\rho Var[Y]\ Var[C] \tag{14}$$

where ρ is the coefficient of correlation between Y and C.

The value of b, b^*, which minimizes the Var[Y(b)] can be found by differentiating (13) with respect to b and is given by

$$b^* = \frac{Cov[Y,C]}{Var[C]} \tag{15}$$

Substituting the above into (12) yields the optimal control variate estimator $Y(b^*)$. The variance of this estimator is then

$$Var[Y(b^*)]=Var[Y]\ (1-\rho^2) \tag{16}$$

Equation (16) indicates the greater the correlation between Y and C, the greater the reduction in variance.

Kleijnen [9] discusses extensions to multiple control variates

$$Y(b) = Y - \sum_{i=1}^{n} b_i(C_i - \gamma_i) \tag{17}$$

where n is the number of control variables.

Law and Kelton [12] present two general methods for obtaining control variables. The first is to use input random variables, such as arrival rates, service rates, and routing probabilities, since their expectations are known and the sign of the correlation with the output may be known. This type of control variate is known as internal or concomitant. Since they are generated by the simulation to obtain the outputs, using them adds little to the cost of the simulation.

A second method for obtaining control variates is to simulate a similar system whose desired performance measure can be analytically computed. This simulation uses the same random numbers as the first simulation to induce positive correlation. The corresponding output of the second simulation can then be used as the control variate. This type of control variate is called an external control variate. The desired outcome is that the output of the second simulation is positively correlated with its counterpart from

the original simulation. Unlike internal control variates the cost of a second simulation is incurred, which in some cases may be prohibitive. Thus the covariance between the outputs will have to be larger than for the internal control variates to make this approach worthwhile.

A third method for obtaining control variates, suggested by Nelson [15], is an amalgam of the internal and external approaches. He suggests simulating the system to obtain the desired performance measures and the means of the input parameters observed during the simulation run. The control variate is derived by substituting these observed input means into a parametric analytical model of a similar system. The mean of this control variate would be derived in a similar fashion, except the known input means, rather than the observed means, would be substituted into the parametric model. Expressed in the linear control variate format, the control estimator of Y would be

$$Y(b) = Y - b\,(A - \zeta) \tag{18}$$

where Y is the crude estimator obtained from the simulation; $A = g(\hat{\bar{X}}_i)$, is the control variate where $\hat{\bar{X}}_i$ is the observed mean of the input X_i driving the simulation that produced Y and $i=1,\ldots c$, where c is the number of input parameters in the parametric model function g; and $\zeta = g(E[\bar{X}_i])$ where $E[\bar{X}_i]$ is the input mean. Equation (18)

need not be unbiased since the expectation of a function is not in general a function of expectations.

From a cost standpoint the analytic approach has an advantage over the true external in that the cost of the second simulation is avoided. The effectiveness of this approach using the Jackson network as the parametric model is the focal point of this research.

Once a control variate method has been selected the problem of specifying the control coefficient, b, must be addressed.

Consider the case where there is only one control variate, C, and (12) is used as the control estimator. Then the optimal value of b, b^*, is expressed in (15). In general Cov[Y,C] and the Var[C] are not known; therefore, b^* needs to be estimated.

Kleijnen [9] presents a method for estimating b^* from the simulation results. He suggests replacing Cov[Y,C] and Var[C] with their sample equivalents. Consider making n independent replications to obtain n independent and identically distributed (iid) observations of Y and C. Then the sample covariance of Y and C, $\hat{\mathrm{Cov}}[Y,C]$, is given by

$$\hat{\mathrm{Cov}}[Y,C]=\frac{1}{n-1}\sum_{i=1}^{n}(Y_i-\bar{Y})(C_i-\bar{C}) \tag{19}$$

and the sample variance of C, $\hat{\mathrm{Var}}[C]$ is

$$\hat{\mathrm{Var}}[C]=\frac{1}{n-1}\sum_{i=1}^{n}(C_i-\bar{C})^2 \tag{20}$$

then the estimator for b, $\hat{b}^*$ is given by

$$b^* = \frac{\hat{Cov}[Y,C]}{\hat{Var}[C]} \tag{21}$$

This produces a final point estimator of Θ

$$Y(\hat{b}^*) = \bar{Y} - \hat{b}^*(\bar{C} - \gamma) \tag{22}$$

It should be noted that $Y(\hat{b}^*)$ may not be unbiased since $\hat{b}^*$ and C are not usually independent, since $\hat{b}^*$ is a function of C as given by (21). The author discusses two techniques, splitting and jackknifing, for reducing the bias of $Y(\hat{b}^*)$.

The case of multiple control variates is addressed by Lavenberg and Welch [10]. The following notation is adopted to rewrite (17) in matrix form. Let $\underline{X}$ be a column vector, and $\underline{X}'$ be its transpose. Then $\underline{C}$ is a column vector of Q control variates and $\underline{\gamma}$ is the mean vector corresponding to C where $\gamma_i = E[C_i]$. Let b be a vector of constants. Then an estimator of Θ is

$$Y(\underline{b}) = Y - \underline{b}'(\underline{C} - \underline{\gamma}) \tag{23}$$

The vector $\underline{b}^*$ which minimizes $Var[Y(\underline{b})]$ is

$$\underline{b}^* = \Sigma_C^{-1}\, \underline{\sigma}_{YC} \tag{24}$$

where Σ_C is the covariance matrix of C and $\underline{\sigma}_{YC}$ is the Q-dimensional vector whose components are the covariances between Y and the C_i, $i=1,\ldots,Q$. This leads to a minimum variance for $Y(\underline{b}^*)$:

$$Var[Y(\underline{b}^*)] = (1-R^2_{YC})\ Var[Y] \tag{25}$$

where

$$R^2_{YC} = \frac{\underline{\sigma}'_{YC}\ \Sigma_C^{-1}\ \underline{\sigma}_{YC}}{Var[Y]} \tag{26}$$

and $(1-R^2_{YC})$ is called the minimum variance ratio. R^2_{YC} is the square of the multiple correlation coefficient between Y and C.

As with the single control variate b^* is unknown and must be estimated. An estimator of b^* is

$$\hat{\underline{b}}^* = \Sigma_C^{-1}\ \underline{\sigma}_{YC} \tag{27}$$

where Σ_C is the sample covariance matrix and $\underline{\sigma}_{YC}$ is the sample covariance vector.

To derive interval estimates the authors consider observations from J statisically independent but otherwise identical runs. Then $\underline{C}$ would be a vector of control variates whose components are the values of $\underline{C}_j$ on the jth replication. Then

$$Y_j(\hat{\underline{b}}^*) = Y_j - \hat{\underline{b}}^*(\underline{C}_j - \underline{\gamma}) \tag{28}$$

and

$$\bar{Y}(\hat{b}^*) = \frac{1}{J} \sum_{j=1}^{J} Y_j(\hat{b}^*) \tag{29}$$

In general $\bar{Y}(\hat{b}^*)$ is not an unbiased estimator of θ and the t-distribution with J-1 degrees of freedom cannot be used to derive the interval estimate. The authors derive confidence intervals for the multiple control case based on the assumption that the vector $(Y, C_1, \ldots, C_Q)$ has a multivariate normal distribution. Under this assumption standard regression techniques can be used to produce $\hat{\mathrm{Var}}[\bar{Y}(\underline{b}^*)]$

$$\frac{\bar{Y}(\hat{b}^*) - \theta}{[\hat{\mathrm{Var}}[\bar{Y}(\hat{b}^*)]]^{.5}} \sim t(J-Q-1) \tag{30}$$

where $t(J-Q-1)$ is the appropriate ordinate from the t-distribution with J-Q-1 degrees of freedom.

In addition the ratio

$$\frac{\mathrm{Var}[\bar{Y}(\hat{\underline{b}}^*)]}{\mathrm{Var}[\bar{Y}]} = \frac{J-2}{J-Q-2}(1 - R^2_{YC}) \tag{31}$$

The above equation indicates that if J, the number of replications , is not large with respect to Q, the number of control variates, the variance reduction produced by $(1-R^2_{YC})$ will be diminished. The authors report experimentation which showed this factor accurately predicted losses in variance reduction.

APPLICATION OF CONTROL VARIATES

The control variate approach was applied to queueing network simulations by Lavenberg, Moeller and Welch [11], and Gaver and Schedler [5]. A summary of these works follows.

Lavenberg et al. [11] considered the application of internal control variates to a broad class of closed networks. These networks allowed priorities, blocking, different customer types and arbitrary service time distributions. Their network consisted of n finite interconnected nodes with one or more servers, and d=1,...,D customer types. A type d event is the departure of a type d customer. Customer routing through the network was controlled by an (nxn) transition probability matrix. The following measures were obtained: $W_i(d)$, the expected queue time for type d customers at node i; $\lambda(d)$, the expected rates at which events occur for type d customers; T(d), the expected time for type d customers to cycle through the network and return to the first node.

The authors experimented with three types of control variates, all of which were internal control variates. The first, work variables, represented the sum of service times for type d customers at node i for type d events in the system. The second, flow variables, represented the fraction of type d events at node i. The third, service variables, represented the sample mean service times for type d customers at node i.

The authors reported substantially larger variance reductions using work variables as opposed to flow or service variables. Experimentation was then limited to work variables. For a network consisting of four to six nodes and one customer type, they report predicted actual variance ratios using six control variates ($Q=6$) of .30 to .85. Predicted actual variance ratios were obtained by multiplying the estimated minimum variance ratios by the theoretical loss factor. Estimated variance ratios ranged from .16 to .77, and are ratios of the variance of the point estimator with work variables to the variance of the crude point estimator. The largest variance reductions for waiting times were achieved at the node having the largest utilization factor.

Wilson and Pritsker [21] performed a similar study using standardized work variables. Work variables are standardized for a specific time period by correcting each variable

observed by its mean and standard deviation. This is performed so the control variates would be asymptotically stable, ensuring efficiency gains are sustained over increasing statistic accumulation intervals. The authors report variance reductions of 20 to 90 percent. They stated their standardized work variables could not be extended to simulations of open and mixed networks.

Gaver and Schedler [5] applied external control variates to a closed network. Their study was the only one found reporting results for external controls. Their network contained two service nodes each offering three different types of services. They allowed for priority service and a mixture of arbitrary and exponential service time distributions. Steady state utilization factors were the performance measures of interest. Their control variate was the utilization obtained from the simulation of a similar but numerically tractable model.

Results were reported for control variate estimators using a control coefficient equal to one and an estimated optimal control coefficient based on (21). For the control coefficient equal to one variance reductions of 51 to 99 percent were achieved with one exception: a node with 99 perecent utilization produced a 29 percent increase in variance. For the estimated optimal control coefficient case variance reductions of 81 to 99 percent were achieved. The

authors note the latter estimates may not be unbiased since b was estimated from the data; however, this bias decreases as the sample size increases. No direct conclusions could be drawn about the relationship of utilization and variance reduction. The results did indicate a trend in which utilization estimates with large sample variances showed the largest variance reductions after the application of control variates.

As stated in Chapter 1 the purpose of this research is to study the effectiveness of Jackson networks as external and analytic control variates for open queueing networks. Figure 3 places this research in the context of previous work in this area.

Control Variates in Queueing Network Simulations

Closed
- Internal Lavenberg et al
- External Gaver and Schedler

Open(this research)
- External
- Analytic

Figure 3: Context of the Research

The study of external and analytical control variates applied to open queueing networks is largely without precedent. The network structures to be tested and the

methodology of carrying out these tests are very experimental. This makes it difficult to predict in advance the suitability of these controls for this class of simulation problems.

CHAPTER III

METHODOLOGY

The primary objective of this research is to investigate the effectiveness of the Jackson model as a control variate for queueing network simulation. Three different network structures were investigated, each meeting the restrictions of the Jackson model with the exception of service time distributions. Service distributions investigated were the exponential, the Weibull, and the uniform. For each network two types of control variates were obtained: the traditional external control variate and the analytic control variate. These controls were obtained to estimate the steady state measures of server utilization factor and customer queue time at each node.

The utilization factor was selected because it serves as an indicator of the level of activity or degree of congestion at a particular node i. The queue time provides the long run waiting time a customer will experience in a given queue (excluding service time), and when applied in Little's formula, $LQ_i=e_i\ WQ_i$, yields the long run queue length. Additionally, the steady state values for the

total time spent at a service node i, W_i, and the number of customers at the node, L_i, can be found by substituting WQ_i and LQ_i into (9) and (11).

TYPES OF CONTROL VARIATES

The classical external control variate requires two simulations. The first simulation is of the network of interest and estimates the utilization factor and queue time for each node. The second simulation is of a Jackson network approximation of the original system. Since the exponential distribution yields the Jackson model itself, external control variates are obtained only for the Weibull and uniform cases. For each of these distributions a second simulation was run with common random numbers using the means of the Weibull and uniform distributions as the parameters of the exponential, and the two desired performance measure estimates were obtained. Using these means and the Jackson model equations, the corresponding steady state measures were obtained analytically. The control variate estimators at each node i (i=1,...,N) for the utilization factor, $RO.C_i$, and queue time, $WQ.C_i$, based on external control variates are given by

$$RO.C_i=RO.S_i-b(RO.E_i-RO(J)_i) \quad (32)$$

$$WQ.C_i=WQ.S_i-b(WQ.E_i-WQ(J)_i) \quad (33)$$

where $RO.S_i$ is the crude estimator of the utilization factor at node i obtained from the simulation; $RO.E_i$ is the external control variate obtained from the second simulation; and $RO(J)_i$ is the analytic value of the steady state Jackson network based on the parameters $\underline{\lambda}$, u_i, $\underline{r}$; $WQ.S_i$, $WQ.E_i$, and $WQ(J)_i$ are defined similarly for the queue times.

The major drawback of this type of control variate is the cost of the second simulation and the associated problem of synchronization. A system is synchronized when a random number used for a purpose, such as arrival or service times, in one system is used for the same purpose in the other systems being compared. The random numbers are those generated from the uniform (0,1) distribution and mapped through an inverse transformation into the desired probability distribution, such as Poisson or exponential. Synchronization tries to solve the problem of insuring that differences between the two simulations are due to model performance and not random number sequences or coding structure. If the systems were not synchronized the comparison of control variate performance might be influenced by the misapplication of random numbers.

In contrast, the analytic control variate requires only simulation of the network of interest. To obtain the analytic control variate additional coding is added to the

simulation program to record the vectors of observed mean arrival rates, $\hat{\underline{\lambda}}$, service rates, $\hat{\underline{u}}$, and the observed fraction of customers routed to the various nodes, $\hat{\underline{r}}$. The analytic control variates at each node i for the utilization factor, $RO(\hat{J})_i$, and queue time, $WQ(\hat{J})_i$, are obtained from the Jackson model equations

$$RO(\hat{J})_i=e_i/(s_i u_i) \tag{34}$$

$$WQ(\hat{J})_i=LQ_i/e_i \tag{35}$$

where e_i is defined in (1) and LQ_i is defined in (8). The analytic Jackson values for the steady state utilization factors and queue times were calculated in the same way as the external control variates. Analytic control variate estimates could then be calculated from

$$RO.C_i=RO.S_i-b(RO(\hat{J})_i-RO(J)_i) \tag{36}$$

$$WQ.C_i=WQ.S_i-b(WQ(\hat{J})_i-WQ(J)_i) \tag{37}$$

Equations (32) and (33) are the same as (36) and (37) with the exception of the control variate terms. In (32) and (33) the control variates $RO.E_i$, and $WQ.E_i$ are obtained from a second simulation of a network modelled as a Jackson network. In (36) and (37) the control variates are

obtained by substituting $\hat{\Lambda}$, $\hat{\underline{u}}$, $\hat{\underline{r}}$ into the Jackson equations.

The tradeoff in using the analytic control variate is the additional code required to obtain $\hat{\Lambda}$, $\hat{\underline{u}}$, $\hat{\underline{r}}$. This additional coding is insignificant relative to the cost of a second simulation. Once the controls or data needed to compute the controls has been obtained the computational effort to obtain the control variate estimates is the same for both type of control variates.

One drawback of the analytic control variate is at high traffic intensities (ρ=.9) it is possible to obtain sample values for $\hat{\Lambda}$, $\hat{\underline{u}}$, $\hat{\underline{r}}$ which violate Jackson model assumptions, specifically $e_i/(s_i u_i)<1$. This does not permit the calculation of an analytic control variate based on the Jackson equations. A possible solution could be to observe the effective arrival rates, $\hat{\underline{e}}$, rather than observing $\hat{\Lambda}$, and $\hat{\underline{r}}$. This approach was employed with one of the networks. Further studies of these two calculation methods is required to determine the benefits and tradeoffs of each method. Another drawback is that, in general, the expectation of a function is not equal to the function of the expectation (e.g. $E[RO(J)_i] \neq RO(J)_i$); however, because it is a method of moments estimator, it is consistent. Therefore using the observed mean arrival and service rates in the Jackson model equations will result in a biased control

variate. A study to determine the severity of this bias is an area open to future reseach, since reduced variance at the expense of significantly large mean squared error is unacceptable.

NETWORKS

To obtain a representative appraisal of the effectiveness of Jackson model control variates, three networks with different structure and complexity were simulated using common random numbers. The first network consists of two nodes, each with its own external arrival process. Customers completing service at each node may be routed to the other node for service or may depart the system entirely.

The second network is a three node tandem, acyclic network. Tandem means the nodes are arranged in series and acyclic means customers will visit each node once. External arrivals occur only at the first node where customers complete service and move to the second and then third nodes for service. Departure from the network occurs only when service is completed at the third node.

The third network consists of four nodes with an external arrival process at the first node. Customers completing service at the first node are routed either to the second or third nodes and then on to the fourth. Customers completing service at the fourth node may be fed back to the first node or depart the system entirely.

Three service time distributions were studied for each of the above networks. The first distribution, the exponential, is the requisite for the Jackson model. It has an infinite tail, is highly variable, and provides analytically tractable performance measures for comparing the control variate estimators. The second distribution, the Weibull, is similar to the exponential. It also has an infinite tail, but it is not as variable as the exponential, as characterized by the coefficient of variation. By setting the Weibull shape parameter to 2 a humped distribution was obtained, thereby providing another reference point to measure the effectiveness of the Jackson controls. The third distribution, the uniform, was selected for its marked difference from the exponential. It has finite range and is considerably less variable. The selection of these distributions provides three references for studying the Jackson controls: the exponential, the requisite for the Jackson model, infinite in the tail, and highly variable; the Weibull, similar to the exponential but humped in our examples; and the uniform, a finite range distribution with considerably less variability.

Other features of the networks studied include traffic intensity and the number of servers at each node. To study the effect of congestion on control variate performance both high, $\rho = .90$, and low, $\rho = .50$, traffic intensities

were studied. Traffic intensity measures the fraction of the systems service capacity being utilized on the average by arriving customers. Traffic intensities close to 1 mean there will rarely be idle servers, so customers will be found backing up into the queues. Queue times and lengths will therefore be larger. Single and multiple servers were studied in each network.

In summary, the basic experiment was to investigate two types of Jackson control variates, external and analytic, for estimating the utilization factors and waiting times in three different queueing networks. The control variates for each network were obtained for three service time distributions: exponential, Weibull, and uniform; and at both high and low traffic intensities. Figure 4 provides an outline of the basic experiment for a given network. Figures 5-7 provide schematics and parameters for each of the three networks.

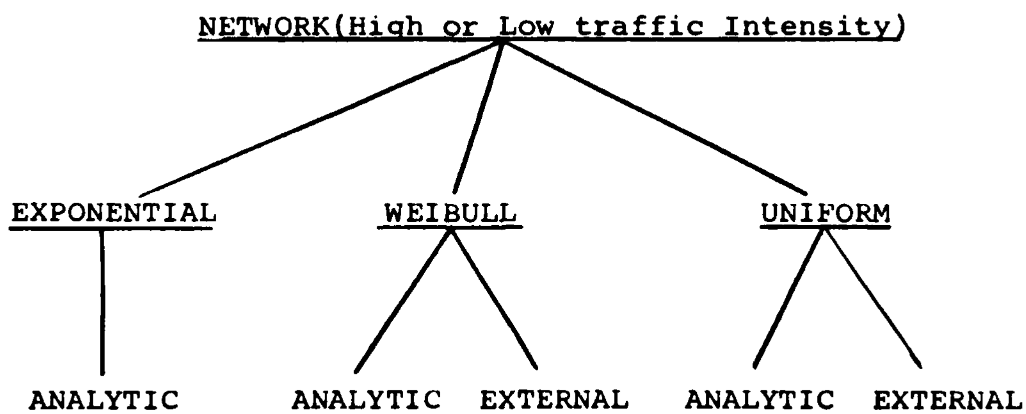
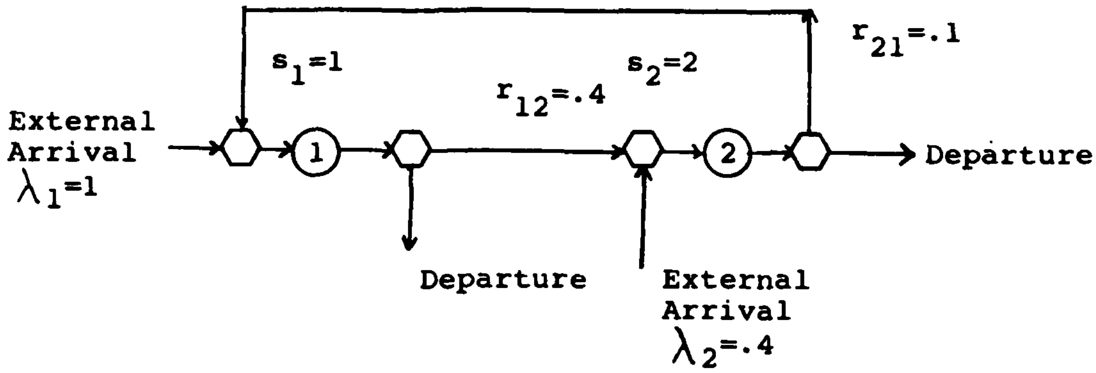


Figure 4: Outline of the Basic Experiment



SERVICE DISTRIBUTIONS

EXPONENTIAL

NODE	MEAN(ρ =.9)	MEAN(ρ =.5)
1	.8308	.4616
2	2.1589	1.2000

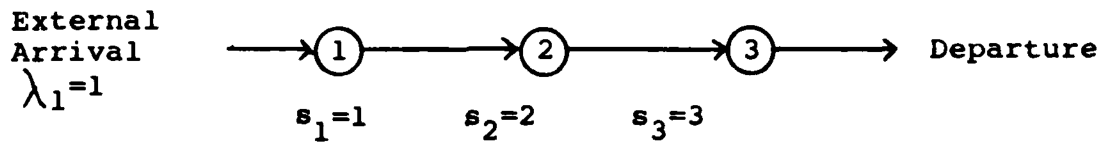
WEIBULL(ALPHA=2)

NODE	BETA(ρ =.9)	BETA(ρ =.5)
1	.9375	.5209
2	2.4371	1.3541

UNIFORM(a,b)

	(ρ =.9)		(ρ =.5)	
NODE	a	b	a	b
1	.4616	1.2000	.3232	.6000
2	1.8196	2.5000	.9000	1.5000

Figure 5: Network I and parameters



SERVICE DISTRIBUTIONS

EXPONENTIAL

NODE	MEAN(ρ =.9)	MEAN(ρ =.5)
1	.9000	.5000
2	1.7999	1.0000
3	2.6998	1.4999

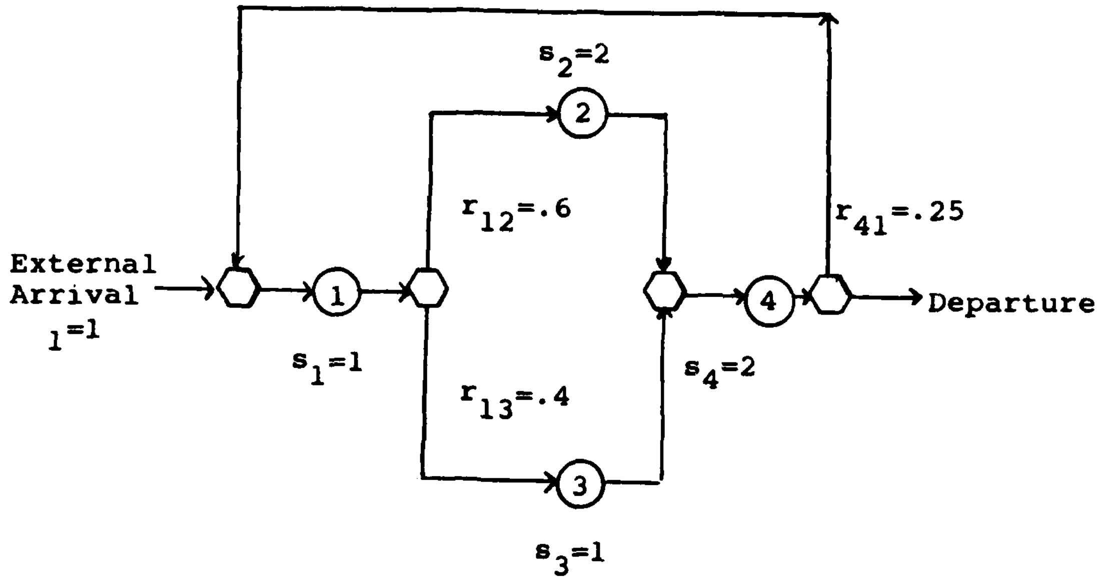
WEIBULL(ALPHA=2)

NODE	BETA(ρ =.9)	BETA(ρ =.5)
1	1.0156	.5642
2	2.0310	1.1282
3	3.0464	1.6925

UNIFORM(a,b)

	(ρ=.9)		(ρ=.5)	
NODE	a	b	a	b
1	.4000	1.4000	.2500	.7500
2	1.3998	2.2000	.7000	1.3000
3	2.1996	3.2000	1.0988	1.9000

Figure 6: Network II and parameters



SERVICE DISTRIBUTIONS

EXPONENTIAL

NODE	MEAN(ρ =.9)	MEAN(ρ =.5)
1	.6750	.3750
2	2.2502	1.2500
3	1.6875	.9376
4	1.3501	.7500

WEIBULL(ALPHA=2)

NODE	BETA(ρ =.9)	BETA(ρ =.5)
1	.7955	.4232
2	2.5392	1.4105
3	1.9042	1.0579
4	1.5234	.8463

UNIFORM(a,b)

	(ρ =.9)		(ρ =.5)	
NODE	a	b	a	b
1	.3600	1.0500	.3000	.4500
2	1.7504	2.7500	.5000	2.0000
3	1.2750	2.1000	.4752	1.4000
4	.9502	1.7500	.2500	1.2500

Figure 7: Network III and parameters

EXPERIMENTAL DESIGN

The basic computational steps required to obtain the analytic control variate estimators are defined by (36) and (37).

Previously it was mentioned that under high traffic intensity conditions it was not always possible to obtain a control variate for a given batch (defined below). In these cases the vectors $\hat{\underline{\lambda}}$, $\hat{\underline{u}}$, $\hat{\underline{r}}$ may produce utilization factors greater than or equal to one, a violation of the Jackson model assumptions. To handle this case the ratio of arrival rate to service rate was set equal to .9999 whenever the utilization factor was greater than or equal to one. This in effect is the use of control variates from all the batches at the expense of introducing some bias into the control estimator.

The discussion pertaining to (21) outlined the procedure for obtaining the estimated optimal control coefficient. The procedure required n independent replications to obtain n iid observations of the crude estimator and its control. From these n values the sample variance and covariance terms of (21) can be calculated.

Since the performance measures of interest are steady state measures, an initial bias period for each replication would have to be deleted. If the initial transient is long, as will be discussed later, the cost of deleting n initial

bias periods becomes excessive. To avoid this costly approach a simulation run consisting of J approximately independent batches of time length t was used instead of the n independent replications.

Following this procedure, the Y and C of equations (19) through (21) are now replaced by $\overline{RO}.S_{ij}$, and $\overline{RO}(\hat{J})_{ij}$, where $\overline{RO}.S_{ij}$ is the batch mean for the crude estimator of the utilization factor at node i in batch j, where $i=1,\ldots,N$ and $j=1,\ldots,J$. $\overline{RO}(\hat{J})_{ij}$ is the analytic control variate derived from $\hat{\underline{\lambda}}$, $\hat{\underline{u}}$, $\hat{\underline{r}}$ in batch j. Let

$$\overline{RO}.S_{i\cdot} = \frac{1}{J} \sum_{j=1}^{J} \overline{RO}.S_{ij} \qquad i=1,\ldots,N \tag{38}$$

$$\overline{RO}(\hat{J})_{i\cdot} = \frac{1}{J} \sum_{j=1}^{J} \overline{RO}(\hat{J})_{ij} \tag{39}$$

$$\hat{Var}[\overline{RO}(\hat{J})_{i}] = \frac{1}{J-1} \sum_{j=1}^{J} (\overline{RO}(\hat{J})_{ij} - \overline{RO}(\hat{J})_{i\cdot})^2 \tag{40}$$

$$\hat{Cov}[\overline{RO}.S_i,\overline{RO}(\hat{J})_i]=\frac{1}{J-1}\sum_{j=1}^{J}(\overline{RO}.S_{ij}-\overline{RO}.S_i.)$$
$$x(\overline{RO}(\hat{J})_{ij}-\overline{RO}(\hat{J})_i.) \tag{41}$$

Using (21) the estimated value for the optimal control coefficient of the utilization factor at node i is

$$\hat{b}^*=\frac{\hat{Cov}[\overline{RO}.S_i,\overline{RO}(\hat{J})_i]}{\hat{Var}[\overline{RO}(\hat{J})_i]} \tag{42}$$

An analagous procedure was followed to obtain $\hat{b}^*$ for WQ.C.

The value of $\hat{b}^*$ computed in (42) can be used to compute the control variate estimate for the run by using

$$RO.C_i=\overline{RO}.S_i.-\hat{b}^*(\overline{RO}(\hat{J})_i.-RO(J)_i) \tag{43}$$

Since the utilization factor is a time persistent performance measure, computing $\overline{RO}.S_i$ from batches of equal time length produces an unbiased estimator assuming each of the $\overline{RO}.S_i$ are identically distributed. This is not the case for the queue time performance measure. WQ is a

discrete performance measure, therefore batching by time produces a random number of customer queue times observed in each of the j batches. The overall mean queue time for a run, $\overline{WQ}.S_{i}.$ is given by

$$\overline{WQ}.S_{i}. = \frac{1}{J} \sum_{j=1}^{J} \overline{WQ}.S_{ij} \tag{44}$$

$$= \frac{1}{J} \sum_{j=1}^{J} \left\{ \frac{1}{d_j} \sum_{n \epsilon B_j} WQ.S_{in} \right\}$$

where B_j is the set of all indices of queue times during $((j-1)t, jt)$, and $d_j = |B_j|$. Therefore

$$\overline{WQ}.S_{i}. \neq \frac{1}{D} \sum_{n=1}^{D} \overline{WQ}.S_{in} \tag{45}$$

where D is the total number of queue times observed in the run. To accommodate the discrete case, the grand mean for all the queue times for the run, $\widetilde{WQ}.S_i$, was used to calculate the control variate estimate. $\widetilde{WQ}.S_i$ is given by

$$\widetilde{WQ}.S_i = \frac{1}{D} \sum_{n=1}^{D} WQ.S_{in} \tag{46}$$

The control variate estimator of the queue time for the run is given by

$$WQ \cdot C_i = \widetilde{WQ} \cdot S_i - \hat{b}^* (\overline{WQ}(\hat{J})_i \cdot - WQ(J)_i) \tag{47}$$

The identical approach is taken to obtain the external control variate estimators. The only difference being substituting $\overline{RO} \cdot E_i$ and $\overline{WQ} \cdot E_i$ for $\overline{RO}(\hat{J})_i$ and $\overline{WQ}(\hat{J})_i$ respectively in (43) and (47). In practice these external controls would be obtained by simulating the network of interest as a Jackson network. Since exponential service yields the Jackson model itself, the values of $RO \cdot E_i$ and $WQ \cdot E_i$ equal $RO \cdot S_i$ and $WQ \cdot S_i$ from the networks with exponential service times.

The batch means approach serves two purposes. First, the J batches per run provide a sequence of observations to compute $\hat{b}^*$ and the control variate estimators. Second, K runs of J batches each can be obtained by simulating a total of K·J batches. This will produce a sequence of K control variate estimates so that the properties of the estimator can be evaluated. The primary design issues with this approach are the batch length t, and the number of batches to be collected within a run, and within the entire experiment.

The number of batches selected for a particular run was based on cost considerations and the loss of variance reduction caused by estimating $\hat{b}^*$. This loss was expressed in (31) as a function of the number of control variates and the number of batches used to estimate $\hat{b}^*$.

While multiple controls are possible, this research studies only a single commensurate control; that is, the corresponding performance measure for the Jackson network. The single control approach was adopted to contain the cost of gathering control variate statistics and to facilitate automation. In addition, if the number of control variates is large with respect to the number of batches, considerable loss in variance reduction will result. Since Q=1 the loss factor, LF, can be expressed as

$$LF = \frac{J-2}{J-3} \tag{48}$$

where J is the number of batches in a run. Table 1 lists various loss factors and their corresponding number of batches.

Based on the above comparison and cost factors, the number of batches, J, was set at 25. The tradeoff of estimating $\hat{b}^*$ was then a 5 percent loss in variance reduction.

The batch length issue centers on choosing a time length, t, large enough to secure approximate independence between the batch means. It is assumed the output sequence

Table 1

Loss Factor Comparison

J	Loss Factors(LF)
10	1.14
15	1.08
20	1.06
25	1.05
30	1.04
50	1.02

of crude estimators and their counterpart control variates is covariance stationary, and the batch length will be large enough so that the resulting batch means will be approximately normally distributed. To select the batch length, t, an independence test given by Fishman [4] was employed for each network to evaluate the independence of queue times at each node.

The results of this testing produced a batch size and corresponding number of batches based on a type I error level of .05. The results for the node in each network requiring the largest number of observations per batch are listed in Table 2.

The number of batches was fixed at 25 for cost and loss factor considerations as previously discussed. Since the effective number of customer arrivals at each network was approximately 1.0, it was assumed that over the long run at least one queue time would be observed per unit of time.

Table 2

Results for Batch Means Independence Test

NETWORK	NODE	NO. BATCHES	SIZE	TOTAL OBS.
1	1	49	64	3136
2	1	23	128	2944
3	4	12	512	6144

This assumption facilitates the conversion from discrete batch size to continuous time batch length. This is done by dividing the total number of observations from Table 2 by 25. Results are listed in Table 3.

Table 3

Selected Batch Lengths

NETWORK	OBS./Batch	Selected Batch Length(time units)
1	125.4	150
2	117.8	200
3	245.8	300

The major concern in deciding the number of runs K, the "macro" replications for computing the point and interval estiamtes, was cost. The CPU time required to simulate the two node network for 10 runs was 1.3 seconds, the time for 20 runs was 1.7 seconds, an approximate 24 perecnt increase in CPU time. This time will also increase with network

size. Given the total simulation requirements of this research and its associated cost, the number of runs, K, for each experiment was fixed at 10.

The performance measures being investigated are steady state means; therefore, a procedure to eliminate the initial transient was employed at the start of each experiment. To approximate the length of the initial transient, a pilot run listing the cumulative mean of the queue time at each node in intervals of 100 time units was executed. The results showed that from empty and idle conditions the build up to steady state was very slow. The system appeared very erratic during the first 10,000 time units before settling down in a more predicable region around the steady state conditions. Therefore a conservative policy of eliminating the statistics collected during the first 10,000 time units after starting from empty and idle conditions was adopted.

The pilot runs indicated that those networks with high feedback tended to reach steady state sooner than those with lesser or no feedback. We can only speculate that the high feedback tends to congest the system sooner, which in turn has a stabilizing effect allowing the system to reach steady state at a faster rate. It should be noted that this effect was studied only at high traffic intensities (ρ =.9) and for exponential service.

To summarize, the simulation of a given network structure involved deleting an initial transient of 10,000 time units and collecting output for 10 runs, each run consisting of 25 batches. In addition to the basic control variate estimators previously described, estimators computed using b=1, and the analytic control variate calculated using $\hat{\lambda}$, $\underline{u}$, $\hat{\underline{r}}$ were also obtained.

The variance reduction achieved through a particular strategy was estimated as follows:

1. Compute the means of both the crude estimator and control variate estimator over the K runs.
2. Compute the variance of the crude and control estimators over the K runs
3. Assuming normality compute confidence intervals for the variance ratio of the control variate estimator to the crude estimator.

CHAPTER IV

RESULTS

The results of the control variate experimentation on each of the three networks are listed in this chapter. Three control variate estimates are reported: the analytic, the modified analytic, and the external. The experimentation was conducted to produce control variate estimates using control coefficients equal to 1 and equal to the estimated optimal control coefficient $\hat{b}^*$. When the control coefficient was set to 1 the variance reductions for the utilization factor estimates were slightly greater than those obtained for $\hat{b}^*$; however, this was not true for the queue time estimates. Variances of these estimates were greatly increased when the control coefficient was set to 1; variance reductions for this measure were achieved only when $\hat{b}^*$ was used. Therefore results are reported only for estimates based on $\hat{b}^*$.

The effectiveness of a particular control variate will be reported in terms of the variance reduction ratio; that is, the ratio of the variances of the control variate estimate to the crude estimate. Ratios greater than 1 indicate

an increase in the estimate's variance. A 90 percent confidence interval is computed for each ratio. Ratios involving the expenditure of computer effort are also possible, but not considered here.

The chapter is divided into three sections corresponding to the three networks studied. Results for each network are reported in the following format: a table listing the steady state Jackson values for each performance measure, a table listing the crude estimator and its variance for a given traffic intensity, and a table for each of the three control variate estimators listing the point estimate, its variance, the variance reduction ratio, and the upper and lower bounds of the 90 percent confidence interval. Variance reduction ratios appear under the ratio column, the lower bound under the L column, and the upper bound under the U column. Estimates with variances less than .00005 are reported as "<.00005".

The simulation was coded using SIMSCRIPT II.5 and all required output written to a file. A FORTRAN program was used to perform the control variate analysis. Samples of both programs are in the Appendix.

RESULTS FOR NETWORK I

Table 4 lists the steady state Jackson values for the utilization factors and queue times of network I depicted in Figure 5.

Table 4

Steady State Jackson Values for Network I.

	$\rho=.9$		$\rho=.5$	
NODE	RO(J)	WQ(J)	RO(J)	WQ(J)
1	.9000	7.4771	.5000	.4616
2	.8999	9.1999	.5000	.4001

The following table lists the crude estimates and their variances for each of the three distributions tested at the high traffic intensity level.

Table 5

Crude Estimates for Network I ($\rho=.9$)

NODE	RO.S	VAR	WQ.S	VAR
		Exponential Service		
1	.8872	.0008	5.9196	2.1802
2	.9076	.0006	9.6711	4.3639
		Weibull Service		
1	.8934	.0005	4.1025	.8210
2	.9033	.0003	4.9167	1.0489
		Uniform Service		
1	.8991	.0002	3.7309	.5230
2	.9023	.0002	3.9522	.4220

Table 6 lists results for the analytic control variate estimates at the high traffic intensity.

Table 6 listed results for analytic control variates based on $\underline{\hat{\lambda}}$, $\underline{\hat{u}}$, and $\underline{\hat{r}}$. Additional experimentation was conducted to determine if other combinations of $\underline{\hat{\lambda}}$, $\underline{\hat{u}}$, and $\underline{\hat{r}}$

Table 6

Analytic Estimates for Network I (ρ=.9)

Exponential Service

NODE	RO.C	VAR	L	RATIO	U
1	.9025	.0001	.0487	.1550	.4929
2	.9130	.0002	.1019	.3240	1.0303

NODE	WQ.C	VAR	L	RATIO	U
1	5.0694	1.1656	.1682	.5350	1.7013
2	8.4098	3.3497	.2420	.7695	2.4470

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.9010	.0001	.0401	.1275	.4055
2	.9034	<.00005	.0471	.1499	.4767

NODE	WQ.C	VAR	L	RATIO	U
1	3.7813	.5469	.2095	.6661	2.1182
2	4.1912	.5425	.1626	.5172	1.6447

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.9023	<.00005	.0718	.2284	.7263
2	.9017	<.00005	.0752	.2392	.7607

NODE	WQ.C	VAR	L	RATIO	U
1	3.3113	.3808	.2290	.7281	2.3154
2	3.4828	.3020	.2250	.7154	2.2750

would produce a better control variate. A pilot run of Network I showed a modified analytic control variate based on $\underline{\hat{\lambda}}$, $\underline{u}$, and $\underline{\hat{r}}$, that is based on the observed mean arrival rates, the input mean service time, and the observed routing probabilities, was the most promising. These modified analytic control variate estimators for the utilization factor and queue time are denoted as RO(M) and WQ(M)

respectively. Results for this modified estimator are reported in Table 7.

Table 7

Modified Estimates for Network I (ρ=.9)

Exponential Service

NODE	RO(M)	VAR	L	RATIO	U
1	.8889	.0005	.1974	.6278	1.9964
2	.9016	.0004	.2145	.6821	2.1691

NODE	WQ(M)	VAR	L	RATIO	U
1	5.6553	1.6770	.2419	.7692	2.4461
2	9.3293	4.8245	.3477	1.1055	3.5155

Weibull Service

NODE	RO(M)	VAR	L	RATIO	U
1	.8936	.0002	.1398	.4445	1.4135
2	.8979	.0002	.1504	.4783	1.5210

NODE	WQ(M)	VAR	L	RATIO	U
1	3.8313	.5795	.2219	.7058	2.2444
2	4.3954	.5804	.1740	.5533	1.7595

Uniform Service

NODE	RO(M)	VAR	L	RATIO	U
1	.8990	<.00005	.0716	.2277	.7241
2	.8970	.0001	.0856	.2722	.8656

NODE	WQ(M)	VAR	L	RATIO	U
1	3.3723	.3727	.2241	.7126	2.2661
2	3.4059	.2790	.2079	.6611	2.1023

Table 8 lists results for the external control in Network I at a traffic intensity of .9.

Tables 9-12 list the same type results for Network I at a traffic intensity of .5.

Table 8

External Estimates for Network I (ρ=.9)

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.9017	<.00005	.0146	.0464	.1476
2	.8990	.0001	.1397	.4443	1.4129

NODE	WQ.C	VAR	L	RATIO	U
1	4.6927	.5279	.2022	.6430	2.0447
2	4.6992	.4128	.1237	.3935	1.2513

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.9016	.0001	.2048	.6514	2.0715
2	.9004	.0001	.1608	.5114	1.6263

NODE	WQ.C	VAR	L	RATIO	U
1	3.9298	.8453	.5083	1.6164	5.1402
2	3.8757	.2829	.2108	.6704	2.1319

Table 9

Crude Estimates for Network I (ρ=.5)

Exponential Service

NODE	RO.S	VAR	WQ.S	VAR
1	.4940	.0003	.4481	.0015
2	.5038	.0002	.3922	.0042

Weibull Service

1	.4965	.0002	.2933	.0003
2	.5026	.0001	.2512	.0004

Uniform Service

1	.4986	.0001	.2420	.0002
2	.5012	.0001	.2064	.0002

Table 10

Analytic Estimates for Network I (ρ=.5)

Exponential Service

NODE	RO.C	VAR	L	RATIO	U
1	.4996	<.00005	.0003	.0009	.0029
2	.4982	<.00005	.0011	.0036	.0114

NODE	WQ.C	VAR	L	RATIO	U
1	.4513	.0005	.1058	.3363	1.0694
2	.3574	.0018	.1318	.4190	1.3324

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.4996	<.00005	.0003	.0008	.0025
2	.4982	<.00005	.0018	.0056	.0178

NODE	WQ.C	VAR	L	RATIO	U
1	.2950	.0002	.2254	.7169	2.2797
2	.2380	.0003	.2083	.6625	2.1068

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.4995	<.00005	.0003	.0011	.0035
2	.4981	<.00005	.0024	.0076	.0242

NODE	WQ.C	VAR	L	RATIO	U
1	.2404	.0002	.2308	.7341	2.3344
2	.2017	.0002	.2558	.8134	2.5866

Table 11

Modified Estimates for Network I (ρ=.5)

Exponential Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4913	.0001	.1152	.3663	1.1648
2	.4955	.0001	.1473	.4685	1.4898

NODE	WQ(M)	VAR	L	RATIO	U
1	.4343	.0007	.1419	.4512	1.4348
2	.3733	.0031	.2279	.7246	2.3042

Weibull Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4940	<.00005	.0796	.2530	.8045
2	.4940	<.00005	.0924	.2939	.9346

NODE	WQ(M)	VAR	L	RATIO	U
1	.2854	.0001	.0796	.2530	.8045
2	.2357	.0003	.1998	.6353	2.0203

Uniform Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4970	<.00005	.0215	.0685	.2178
2	.4942	<.00005	.0106	.0338	.1075

NODE	WQ(M)	VAR	L	RATIO	U
1	.2362	.0002	.2685	.8538	2.7151
2	.1958	.0002	.2869	.9124	2.9014

Table 12

External Estimates for Network I (ρ=.5)

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.5007	<.00005	.0392	.1246	.3962
2	.5007	<.00005	.0379	.1206	.3835

NODE	WQ.C	VAR	L	RATIO	U
1	.3025	.0004	.3276	1.0418	3.3129
2	.2552	.0005	.3473	1.1044	3.5120

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.5008	<.00005	.2564	.8155	2.5933
2	.4998	<.00005	.1793	.5703	1.8136

NODE	WQ.C	VAR	L	RATIO	U
1	.2492	.0006	.8307	2.6416	8.4003
2	.2061	.0002	.2659	.8455	2.6887

RESULTS FOR NETWORK II

Table 13 lists the steady state Jackson results for the utilization factors and queue times of Network II depicted in Figure 6.

Table 13

Steady State Jackson Values for Network II.

	ρ =.9		ρ=.5	
NODE	RO(J)	WQ(J)	RO(J)	WQ(J)
1	.9001	8.1089	.5000	.5000
2	.8999	7.6667	.5000	.3333
3	.9001	7.3466	.5000	.2368

Tables 14-17 list results for Network II at a traffic intensity of .9.

Table 14

Crude Estimates for Network II (ρ=.9)

Exponential Service

NODE	RO.S	VAR	WQ.S	VAR
1	.8872	.0005	6.3363	1.1857
2	.8947	.0001	7.3719	4.4346
3	.8906	.0004	6.1086	3.0807
		Weibull Service		
1	.8941	.0002	4.2966	.4055
2	.8939	.0001	2.9552	.4286
3	.8918	.0002	2.2526	.2985
		Uniform Service		
1	.8964	.0001	4.1722	.3479
2	.8946	.0001	1.0114	.0388
3	.8950	.0001	.5184	.0072

Tables 18-21 list results for Network II at a traffic intensity of .5.

Table 15

Analytic Estimates for Network II (ρ=.9)

Exponential Service

NODE	RO.C	VAR	L	RATIO	U
1	.9008	<.00005	.0239	.0759	.2414
2	.9010	<.00005	.1134	.3607	1.1470
3	.8985	.0001	.1013	.3222	1.0246

NODE	WQ.C	VAR	L	RATIO	U
1	5.6212	.2973	.0788	.2507	.7972
2	6.4935	1.8028	.1278	.4065	1.2927
3	5.6702	2.5672	.2608	.8295	2.6378

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.8997	<.00005	.0197	.0625	.1988
2	.8999	<.00005	.0398	.1267	.4029
3	.8978	<.00005	.0619	.1968	.6258

NODE	WQ.C	VAR	L	RATIO	U
1	3.8640	.1924	.1492	.4744	1.5086
2	2.7459	.3320	.2436	.7747	2.4635
3	2.1985	.2648	.2790	.8872	2.8213

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.9015	<.00005	.0477	.1516	.4821
2	.8995	<.00005	.0371	.1181	.3756
3	.8996	<.00005	.0481	.1530	.4865

NODE	WQ.C	VAR	L	RATIO	U
1	3.8384	.4355	.1775	.5645	1.7951
2	.9515	.0314	.2549	.8105	2.5774
3	.4910	.0054	.2373	.7545	2.3993

Table 16

Modified Estimates for Network II (ρ=.9)

Exponential Service

NODE	RO(M)	VAR	L	RATIO	U
1	.8805	.0003	.2214	.7041	2.2390
2	.8971	.0001	.3334	1.0602	3.3714
3	.8917	.0004	.2730	.8682	2.7609

NODE	WQ(M)	VAR	L	RATIO	U
1	6.1244	1.1639	.3087	.9816	3.1215
2	7.1038	3.7865	.2685	.8539	2.7154
3	6.0812	3.4835	.3539	1.1255	3.5791

Weibull Service

NODE	RO(M)	VAR	L	RATIO	U
1	.8932	.0001	.1515	.4818	1.5321
2	.8961	<.00005	.2073	.6592	2.0963
3	.8934	.0001	.1563	.4971	1.5808

NODE	WQ(M)	VAR	L	RATIO	U
1	4.0569	.3096	.2401	.7634	2.4276
2	2.8120	.3552	.2607	.8289	2.6359
3	2.2232	.2692	.2836	.9020	2.8684

Uniform Service

NODE	RO(M)	VAR	L	RATIO	U
1	.8988	<.00005	.1184	.3766	1.1976
2	.8965	<.00005	.0599	.1906	.6061
3	.8970	<.00005	.0476	.1513	.4811

NODE	WQ(M)	VAR	L	RATIO	U
1	3.9215	.2612	.2361	.7507	2.3872
2	.9486	.0314	.2549	.8105	2.5774
3	.4953	.0056	.2435	.7743	2.4623

Table 17

External Estimates for Network II (ρ=.9)

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.9009	<.00005	.0195	.0619	.1968
2	.8976	<.00005	.1520	.4835	1.5375
3	.8972	<.00005	.0265	.0843	.2681

NODE	WQ.C	VAR	L	RATIO	U
1	5.2104	.4523	.3508	1.1154	3.5470
2	3.1890	.1155	.0847	.2694	.8567
3	2.5559	.1728	.1821	.5790	1.8412

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.9003	.0001	.2906	.9242	2.9390
2	.8965	.0001	.3332	1.0596	3.3695
3	.8968	<.00005	.1527	.4855	1.5439

NODE	WQ.C	VAR	L	RATIO	U
1	4.4618	.3898	.3523	1.1203	3.5626
2	1.0130	.0319	.2589	.8232	2.6178
3	.5032	.0074	.3258	1.0359	3.2942

Table 18

Crude Estimates for Network II (ρ=.5)

Exponential Service

NODE	RO.S	VAR	WQ.S	VAR
1	.4931	.0001	.4800	.0012
2	.4973	<.00005	.3472	.0006
3	.4947	.0001	.2278	.0006

Weibull Service

1	.4955	.0001	.3114	.0004
2	.4971	<.00005	.1473	.0001
3	.4958	.0001	.0984	.0001

Uniform Service

1	.4980	<.00005	.2763	.0002
2	.4972	<.00005	.0561	<.00005
3	.4975	<.00005	.0320	<.00005

Table 19

Analytic Estimates for Network II (ρ=.5)

Exponential Service

NODE	RO.C	VAR	L	RATIO	U
1	.4999	<.00005	.0005	.0016	.0051
2	.4997	<.00005	.0064	.0204	.0649
3	.4996	<.00005	.0028	.0090	.0286

NODE	WQ.C	VAR	L	RATIO	U
1	.4829	.0009	.2191	.6968	2.2158
2	.3550	.0004	.1975	.6280	1.9970
3	.2299	.0014	.7819	2.4865	7.9071

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.4999	<.00005	.0012	.0037	.0118
2	.4999	<.00005	.0049	.0156	.0496
3	.4997	<.00005	.0028	.0089	.0283

NODE	WQ.C	VAR	L	RATIO	U
1	.3136	.0005	.3875	1.2321	3.9181
2	.1457	.0001	.2642	.8402	2.6718
3	.0995	.0001	.4986	1.5854	5.0416

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.5000	<.00005	.0032	.0101	.0321
2	.5000	<.00005	.0040	.0126	.0401
3	.5000	<.00005	.0028	.0089	.0283

NODE	WQ.C	VAR	L	RATIO	U
1	.2759	.0002	.2534	.8057	2.5621
2	.0561	<.00005	.3201	1.0179	3.2369
3	.0320	<.00005	.3655	1.1624	3.6964

Table 20

Modified Estimates for Network II (ρ=.5)

Exponential Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4932	.0001	.2045	.6503	2.0680
2	.4981	.0001	.3856	1.2263	3.8996
3	.4950	.0001	.1539	.4894	1.5563

NODE	WQ(M)	VAR	L	RATIO	U
1	.4739	.0009	.2403	.7643	2.4305
2	.3413	.0005	.2904	.9234	2.9364
3	.2262	.0007	.3731	1.1863	3.7724

Weibull Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4957	<.00005	.1272	.4045	1.2863
2	.4976	<.00005	.2222	.7065	2.2467
3	.4961	<.00005	.0743	.2362	.7511

NODE	WQ(M)	VAR	L	RATIO	U
1	.3080	.0003	.2426	.7716	2.4537
2	.1448	.0001	.3964	1.2604	4.0081
3	.0971	.0001	.3494	1.1110	3.5330

Uniform Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4984	<.00005	.0892	.2837	.9022
2	.4975	<.00005	.0153	.0486	.1545
3	.4979	<.00005	.0205	.0653	.2077

NODE	WQ(M)	VAR	L	RATIO	U
1	.2473	.0003	.3616	1.1498	3.6564
2	.0554	<.00005	.2763	.8786	2.7939
3	.0315	<.00005	.3875	1.2324	3.9190

Table 21

External Estimates for Network II (ρ=.5)

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.5004	<.00005	.0236	.0751	.2388
2	.4990	<.00005	.0792	.2519	.8010
3	.4996	<.00005	.0109	.0346	.1100

NODE	WQ.C	VAR	L	RATIO	U
1	.3225	.0002	.1824	.5801	1.8447
2	.1449	<.00005	.0418	.1330	.4229
3	.1027	<.00005	.1038	.3302	1.0500

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.5002	<.00005	.3651	1.1610	3.6920
2	.4983	<.00005	.3060	.9730	3.0941
3	.4994	<.00005	.0821	.2611	.8303

NODE	WQ.C	VAR	L	RATIO	U
1	.2805	.0004	.5099	1.6216	5.1567
2	.0560	<.00005	.2915	.9269	2.9475
3	.0319	<.00005	.3700	1.1766	3.7416

RESULTS FOR NETWORK III

Table 22 lists the steady state Jackson values for the utilization factors and queue times of Network III depicted in Figure 7.

Table 22

Steady State Jackson Values for Network III.

	ρ=.9		ρ=.5	
NODE	RO(J)	WQ(J)	RO(J)	WQ(J)
1	.9000	6.0787	.5000	.3750
2	.9001	9.6031	.5000	.4167
3	.9000	15.1853	.5000	.9377
4	.9000	5.7589	.5000	.2500

Tables 23-26 list the results for Network III at a traffic intensity of .9.

Table 23

Crude Estimates for Network III (ρ=.9)

Exponential Service

NODE	RO.S	VAR	WQ.S	VAR
1	.8912	.0005	6.0750	7.5294
2	.8999	.0003	9.0400	8.1951
3	.8874	.0006	13.1571	12.4821
4	.8972	.0003	5.4977	.5474
		Weibull Service		
1	.9320	.0003	6.5081	6.2433
2	.8943	.0001	4.3736	.7746
3	.8943	.0003	7.9743	4.4438
4	.8963	.0001	1.9755	.0951
		Uniform Service		
1	.9353	.0002	6.0980	7.2565
2	.8939	.0002	2.8475	.2466
3	.8946	.0004	5.3534	1.0206
4	.8935	.0002	.5270	.0056

Tables 27-30 list results for Network III at a traffic intensity of .5.

Table 24

Analytic Estimates for Network III (ρ=.9)

Exponential Service

NODE	RO.C	VAR	L	RATIO	U
1	.8989	.0001	.0382	.1215	.3864
2	.9053	<.00005	.0567	.1804	.5737
3	.9044	.0001	.0642	.2041	.6490
4	.9019	.0001	.1107	.3519	1.1190

NODE	WQ.C	VAR	L	RATIO	U
1	5.4466	4.2202	.1763	.5605	1.7824
2	7.8507	4.6498	.1784	.5674	1.8043
3	10.8947	6.9876	.1760	.5598	1.7802
4	5.0025	.7746	.4450	1.4150	4.4997

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.9132	.0001	.0606	.1927	.6128
2	.9007	<.00005	.0723	.2300	.7314
3	.9047	.0001	.0896	.2850	.9063
4	.8990	<.00005	.1021	.3248	1.0329

NODE	WQ.C	VAR	L	RATIO	U
1	5.7123	4.1207	.2075	.6600	2.0988
2	4.0249	.5064	.2056	.6537	2.0788
3	7.2919	3.0851	.2185	.6948	2.2095
4	1.8792	.0878	.2903	.9233	2.9361

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.9143	.0001	.1686	.5361	1.7048
2	.9019	<.00005	.0261	.0829	.2636
3	.9043	.0001	.0464	.1476	.4694
4	.8977	<.00005	.0772	.2455	.7807

NODE	WQ.C	VAR	L	RATIO	U
1	5.2239	4.9088	.2127	.6765	2.1513
2	2.5843	.1150	.1466	.4663	1.4828
3	4.5992	.6964	.2146	.6823	2.1697
4	.5125	.0053	.2977	.9466	3.0102

Table 25

Modified Estimates for Network III (ρ=.9)

Exponential Service

NODE	RO(M)	VAR	L	RATIO	U
1	.8927	.0002	.1575	.5008	1.5925
2	.9021	.0001	.1756	.5583	1.7754
3	.8927	.0003	.1860	.5916	1.8813
4	.8999	.0003	.2756	.8763	2.7866

NODE	WQ(M)	VAR	L	RATIO	U
1	5.7936	8.6035	.3593	1.1426	3.6335
2	8.6201	6.8833	.2641	.8399	2.6709
3	12.6169	10.2152	.2574	.8184	2.6029
4	5.3743	.4023	.2311	.7350	2.3373

Weibull Service

NODE	RO(M)	VAR	L	RATIO	U
1	.9340	.0001	.1216	.3867	1.2297
2	.8985	<.00005	.0726	.2308	.7339
3	.8973	.0001	.0997	.3169	1.0077
4	.8991	.0001	.1627	.5173	1.6450

NODE	WQ(M)	VAR	L	RATIO	U
1	6.1046	5.8130	.2926	.9306	2.9593
2	4.1846	.6406	.2601	.8271	2.6302
3	7.2838	2.8174	.1995	.6345	2.0177
4	1.9310	.0734	.2429	.7724	2.4562

Uniform Service

NODE	RO(M)	VAR	L	RATIO	U
1	.9387	.0001	.0798	.2539	.8074
2	.8991	<.00005	.0322	.1024	.3256
3	.9015	.0001	.0516	.1642	.5222
4	.8961	.0001	.0945	.3004	.9553

NODE	WQ(M)	VAR	L	RATIO	U
1	5.6317	4.3992	.1907	.6063	1.9280
2	2.6042	.1369	.1746	.5551	1.7652
3	4.6425	.6922	.2133	.6782	2.1567
4	.5123	.0054	.3068	.9755	3.1021

Table 26

External Estimates for Network III (ρ=.9)

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.9374	.0001	.1164	.3702	1.1772
2	.8940	.0001	.2039	.6483	2.0616
3	.8977	.0001	.1371	.4361	1.3868
4	.8976	.0001	.1537	.4888	1.5544

NODE	WQ.C	VAR	L	RATIO	U
1	6.6310	1.4742	.0742	.2361	.7508
2	4.5533	.7773	.3156	1.0036	3.1914
3	8.2056	4.5019	.3186	1.0131	3.2217
4	2.0323	.0617	.2041	.6490	2.0638

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.9393	.0002	.2631	.8367	2.6607
2	.8948	.0002	.2281	.7255	2.3071
3	.8969	.0003	.2488	.7912	2.5160
4	.8957	.0002	.3649	1.1603	3.6898

NODE	WQ.C	VAR	L	RATIO	U
1	6.0083	2.5200	.1177	.3743	1.1903
2	3.0071	.3502	.4466	1.4202	4.5162
3	5.4661	1.0297	.3173	1.0089	3.2083
4	.5298	.0062	.3503	1.1141	3.5428

Table 27

Crude Estimates for Network III (ρ=.5)

Exponential Service

NODE	RO.S	VAR	WQ.S	VAR
1	.4942	.0002	.3677	.0009
2	.4969	.0001	.4379	.0041
3	.4957	.0001	.9249	.0074
4	.4976	.0001	.2400	.0003

Weibull Service

NODE	RO.S	VAR	WQ.S	VAR
1	.4970	.0001	.2373	.0001
2	.4976	.0001	.2508	.0007
3	.4983	.0001	.5540	.0025
4	.4990	.0001	.1063	<.00005

Uniform Service

NODE	RO.S	VAR	WQ.S	VAR
1	.4976	.0001	.1867	.0001
2	.4952	.0001	.1928	.0003
3	.5007	.0001	.4486	.0006
4	.4967	.0001	.0731	<.00005

Table 28

Analytic Estimates for Network III (ρ=.5)

Exponential Service

NODE	RO.C	VAR	L	RATIO	U
1	.4995	<.00005	.0005	.0015	.0048
2	.5003	<.00005	.0008	.0025	.0080
3	.5014	<.00005	.0019	.0061	.0194
4	.4996	<.00005	.0023	.0073	.0232

NODE	WQ.C	VAR	L	RATIO	U
1	.3726	.0003	.0926	.2945	.9365
2	.4242	.0027	.2036	.6476	2.0594
3	.9284	.0067	.2839	.9028	2.8709
4	.2363	.0001	.1412	.4490	1.4278

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.4995	<.00005	.0012	.0038	.0121
2	.5005	<.00005	.0007	.0022	.0070
3	.5013	<.00005	.0007	.0022	.0070
4	.4995	<.00005	.0012	.0038	.0121

NODE	WQ.C	VAR	L	RATIO	U
1	.2390	.0001	.1336	.4250	1.3515
2	.2498	.0005	.2347	.7462	2.3729
3	.5518	.0011	.1355	.4310	1.3706
4	.1055	<.00005	.1779	.5656	1.7986

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.4995	<.00005	.0007	.0022	.0070
2	.5002	<.00005	.0005	.0017	.0054
3	.5015	<.00005	.0005	.0017	.0054
4	.4996	<.00005	.0002	.0006	.0019

NODE	WQ.C	VAR	L	RATIO	U
1	.1872	.0001	.1245	.3959	1.2590
2	.1929	.0001	.1413	.4492	1.4285
3	.4442	.0003	.1419	.4513	1.4351
4	.0733	<.00005	.1222	.3886	1.2357

Table 29

Modified Estimates for Network III (ρ=.5)

Exponential Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4956	.0001	.0993	.3159	1.0046
2	.4992	<.00005	.0154	.0491	.1561
3	.4946	<.00005	.1109	.3527	1.1216
4	.4999	.0001	.2875	.9142	2.9072

NODE	WQ(M)	VAR	L	RATIO	U
1	.3673	.0006	.2097	.6668	2.1204
2	.4308	.0023	.1716	.5458	1.7356
3	.9149	.0065	.2779	.8837	2.8102
4	.2413	.0005	.4626	1.4712	4.6784

Weibull Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4967	<.00005	.0479	.1522	.4840
2	.4984	<.00005	.0096	.0306	.0973
3	.4960	<.00005	.0430	.1367	.4347
4	.4991	<.00005	.0755	.2402	.7638

NODE	WQ(M)	VAR	L	RATIO	U
1	.2354	.0001	.1958	.6228	1.9805
2	.2479	.0006	.2623	.8341	2.6524
3	.5379	.0013	.1661	.5283	1.6800
4	.1052	<.00005	.3547	1.1280	3.5870

Uniform Service

NODE	RO(M)	VAR	L	RATIO	U
1	.4986	<.00005	.0039	.0123	.0391
2	.4975	<.00005	.0103	.0326	.1037
3	.4993	<.00005	.0088	.0281	.0894
4	.4974	<.00005	.0279	.0886	.2817

NODE	WQ(M)	VAR	L	RATIO	U
1	.1862	<.00005	.1108	.3525	1.1210
2	.1893	.0001	.1394	.4432	1.4094
3	.4400	.0002	.1148	.3651	1.1610
4	.0722	<.00005	.0711	.2260	.7187

Table 30

External Estimates for Network III (ρ=.5)

Weibull Service

NODE	RO.C	VAR	L	RATIO	U
1	.5011	<.00005	.0708	.2253	.7165
2	.4992	<.00005	.0795	.2529	.8042
3	.4984	<.00005	.2362	.7510	2.3882
4	.4998	<.00005	.1035	.3292	1.0469

NODE	WQ.C	VAR	L	RATIO	U
1	.2398	<.00005	.1007	.3201	1.0179
2	.2538	.0009	.4155	1.3212	4.2014
3	.5573	.0041	.5073	1.6132	5.1300
4	.1086	<.00005	.2761	.8779	2.7917

Uniform Service

NODE	RO.C	VAR	L	RATIO	U
1	.5008	<.00005	.1669	.5308	1.6879
2	.4965	.0001	.2167	.6890	2.1910
3	.5014	.0001	.2286	.7270	2.3119
4	.4966	.0001	.2325	.7393	2.3510

NODE	WQ.C	VAR	L	RATIO	U
1	.1890	.0001	.2299	.7312	2.3252
2	.1938	.0002	.2526	.8032	2.5542
3	.4529	.0004	.2352	.7478	2.3780
4	.0740	<.00005	.4015	1.2768	4.0602

CHAPTER V

CONCLUSIONS

The purpose of this research was to study the application of Jackson networks as control variates in queueing simulations in order to make some general conclusions about their effectiveness for variance reduction. These conclusions hold their importance in that they add to the store of prior knowldge an analyst can draw on in deciding the appropriate variance reduction technique. Also, these results indicate whether or not continued research in this area is warranted.

The results of this study indicate the potential of analytic controls based on Jackson networks to produce variance reductions in utilization factor estimates. Jackson based controls for the queue time estimates were not as effective as the utilization controls in producing variance reductions. In each network studied the queue time control variates produced little or no variance reduction and indicated the potential to increase this estimate's variance. In some cases these controls could increase the control estimate's variance up to eight times that of the

crude estimate's variance. The analytic controls for the utilization factor showed more promise.

In Network I the analytic controls for the utilization factor produced variance reductions in the range of 68 to 88 percent for traffic intensities of .9, and approximately 99 percent for traffic intensities of .5. The modified analytic controls produced variance reductions of approximately 75 percent only for the uniform service time case. Performance in the exponential and Weibull cases indicated the potential to add variance to the estimate. External controls for the utilization factor were poor and again indicated the potential to add variance.

Analytic controls for the utilization factor in Network II produced consistent variance reductions for the Weibull and uniform service cases. Reductions for these controls ranged from 80 to 94 percent at traffic intensities of .9, and approximately 98 percent for traffic intensities of .5. The modified analytic controls performed well only for the uniform service case at traffic intensities of .5. Here the reductions ranged from 71 to 93 percent. External controls were generally poor with the exception of the Weibull service case at the .5 traffic intensity level where the v riance reductions ranged from 74 to 96 percent.

Network II was structured so that each service node would experience the same effective arrival rate in steady

state conditions. The service rate was the same at each node, but the number of servers was varied from one to three (see Figure 6). The purpose was to observe the impact of the number of servers on control variate performance. The results did not indicate any observable connection between the number of servers and control variate performance.

In Network III the analytic controls for the utilization factor at the .9 traffic intensity level showed modest performance. Their performance at the .5 traffic intensity level was greatly improved producing variance reductions of approximatley 99 percent. For modified analytic controls at the .9 traffic intensity level, variance reductions were acceptable only in the uniform service case where the reductions ranged from 70 to 90 percent. At the .5 traffic level intensity these controls showed good performance for both the Weibull and uniform service cases producing reductions in the range of 76 to 98 percent. Performance of the external controls was generally poor.

The results did show the analytic control variates for the utilization factor worked well at the .5 traffic intensity level. The same statement could not be made about the queue time controls since their performance was so erratic at both traffic intensity levels studied. No conclusive statement could be made concerning the impact of the service distribution on control variate performance.

The Jackson based analytic controls indicate promise as effective control variates for the utilization factor. One possible explanation for the difference in performance between the utilization factor and queue time controls may be suggested by the form of these two performance measures. The utilization factor is a ratio based on effective arrival and service rates. The queue time measure has a more complex form incorporating the probability distribution of the number of customers at a service node and the fraction of the customer load carried by the servers; see (8) and (10). This may suggest the variance of the queue time or similar measure may be too complex to be fully captured by the control variate approach. This should not preclude future research in the application of control variates in queueing network simulations. One approach may may be to observe different forms of $\hat{\Lambda}$, $\hat{\underline{u}}$, and $\hat{\underline{r}}$ to obtain the control variates, such as observing $p_i(0)$ of (10) directly from the simulation rather than computing it from $\hat{\Lambda}$, $\hat{\underline{u}}$, and $\hat{\underline{r}}$. Another approach worth considering is to search for models which provide close approximations for the performance measures of interest as opposed to the exact analytical value provided by the Jackson model. The use of approximation models may very well broaden the class of queueing networks receptive to the control variate approach. Whitt [20] has investigated the use of open

networks to approximate the performance of closed queueing networks. The opportunity of expanding the use of control variates in open systems as approximations for closed networks would be enhanced by further research in this area.

Another possible approach to improving control variate performance is to obtain a more precise estimate of the control coefficient b. One way to accomplish this would be to increase the number of batches in a replication. A pilot run increasing the number of batches from 25 to 50 was performed on Network I with Weibull service at the .9 traffic intensity level. Little improvement was noted in the performance of the utilization controls; however, considerable improvement was seen in the queue time controls. Variance reductions doubled for the analytic, modified analytic, and external controls. This may suggest the poor performance of the Jackson based queue time controls is not solely due to the Jackson model. The ability or inability to accurately estimate b^* may have a major impact on control variate performance for these systems. The methodology for estimating the control coefficient is open to further study.

Other sources which may explain the poor performance of the control variates lie in the methodology of this study. In order to obtain an interval estimate and a value for $\hat{b}^*$ the batch means approach was employed rather than running a

series of independent replications. This is not an uncommon practice and is employed to reduce the cost of the simulation. The batch means approach, however, produces only approximate independence between the batches. Determining the batch size is critical to this independence and is complicated by network structure. A batch size may work well for one particular node and not as well for the remaining service nodes. Further study is needed to determine the usefulness of the batch means approach in this methodology. Another source for error is the initial bias. These networks tended to have long and erratic initial bias periods. This study took a fairly conservative approach in deleting this bias; however, further study of the initial bias in networks is needed to improve the application of control variates for steady state analyses.

The results of this study do highlight the potential of analytic control variates in simulation. Depending on the parametric model selected to serve as the basis for the control, the effort required to obtain variance reduction would be small compared to reducing the variance through additional run time or the second simulation required by external controls. This holds considerable promise for automating or incorporating the analytic control approach in existing simulation languages. In software designed for a specific user this approach could be incorporated by the

addition of a statistical collection mechanism and a routine to derive the controls from these statistics. The benefit of this endeavor would be to avail a wider range of variance reduction techniques to the user community and enhance the analysis provided through computer simulation.

Appendix A

COMPUTER CODE

```
//    JOB ,
//    REGION=768K,TIME=7
/*JOBPARM   LINES=5000,V=S,DISKIO=5000
//    EXEC  SIM93CG,TIME.GO=6
//CMP.SYSIN    DD  *
''ANTHONY P. SHARON    ADVISOR: DR. BARRY L. NELSON
''DEPT: ISE         THESIS RESEARCH
''APPLICATION OF JACKSON NETWORKS AS EXTERNAL CV
''FOR QUEUEING SIMULATION      SYSTEM: 2    NODES
''ARRIVAL:EXPONENTIAL         SERVICE:EXPONENTIAL
''BATCH LENGTH: 150            INITIAL DELETION:10000
''NO. OF MACROS: 10              BATCHES PER MACRO: 25
''ATIM(I)                   INTERARRIVAL TIME AT NODE I
''BR(I)                     INPUT BRANCHING PROBABILITIES FROM
''                          NODE I TO NODE J
''BUSY(I)                   NO. OF BUSY SERVERS AT NODE I
''CUST                      CUSTOMER
''LAMBDA(I)                 EXTERNAL ARRIVAL RATE AT NODE I
''MU(I)                     SERVICE RATE AT NODE I
''NODT                      ENTRY TIME AT A NODE
''O.R(I,J)                  OBSERVED BRANCHING FROM NODE I TO J
''R(I,J)                    COMPARISON ROUTING MATRIX BASED ON BR(I,J)
''RCUST(I,J)                NO. OF CUSTOMERS ROUTED FROM NODE I TO J
''S(I)                      NO. OF SERVERS AT NODE I
''STIM(I)                   SERVICE TIME AT NODE I
''TCUST(I)                  NO. OF CUSTOMERS COMPLETING SERVICE
''                          AT NODE I
''WTIM(I)                   QUEUE TIME AT NODE I
PREAMBLE   LAST COLUMN IS 72''
EVENT NOTICES INCLUDE RESET, OUTPUT
      EVERY ARRV HAS A NODE.A
          DEFINE NODE.A AS AN INTEGER VARIABLE
      EVERY EOS HAS A CUST.E, A NODE.E
          DEFINE CUST.E, NODE.E AS INTEGER VARIABLES
PERMANENT ENTITIES
      EVERY NODE HAS AN ATIM, A BUSY, A LAMBDA, A MU, AN S,
                  AN AWQ,  AN STIM, A TCUST, A WTIM AND OWNS A QUEUE
          DEFINE BUSY, S, TCUST AS INTEGER VARIABLES
TEMPORARY ENTITIES
      EVERY CUST HAS AN NODT AND MAY BELONG TO THE QUEUE
DEFINE RCUST AS A 2-DIMENSIONAL INTEGER ARRAY
DEFINE BR, O.R, R AS 2-DIMENSIONAL ARRAYS
DEFINE NM,C AS  VARIABLES
ACCUMULATE A.BUSY AS THE MEAN OF BUSY
TALLY A.WQ AS THE MEAN OF WTIM
TALLY A.AR AS THE MEAN OF ATIM
TALLY A.SR AS THE MEAN OF STIM
TALLY G.AWQ AS THE MEAN OF AWQ
END    ''PREAMBLE
MAIN
DEFINE I AS AN INTEGER VARIABLE
LET NM=1
CREATE EVERY NODE (2) ''NO. OF NODES
RESERVE BR(*,*), R(*,*), RCUST(*,*), O.R(*,*) AS  2 BY 2
READ BUSY, LAMBDA, S
START NEW RECORD
READ MU
START NEW RECORD
READ BR
```

```
FOR I=1 TO 2 , DO            ''NO. OF NODES
    FOR J= 1 TO 2, DO       ''NO.OF NODES
          LET CR= BR(I,J) + CR
          LET R(I,J)= CR
     LOOP
     LET CR=0
LOOP
PRINT 1 LINE THUS
               ECHO INPUT
SKIP 2 LINES
FOR I=1 TO 2, DO                ''NO.OF NODES
    PRINT 5 LINES WITH I, LAMBDA(I), MU(I),
                       BUSY(I), S(I) THUS
       INPUT VALUES FOR NODE  *
   ARRIVAL RATE:        **.****
   SERVICE RATE:        **.****
   NO. BUSY SERVERS:    **
   NO. OF SERVERS:      **
     SKIP 2 LINES
LOOP
SKIP 2 LINES
LIST BR
SKIP 1 LINE
LIST R
SKIP 2 LINES
''SCHEDULE ARRIVAL FOR NODES WITH EXTERNAL ARRIVALS
   FOR I=1 TO 2, DO
       LET NODE= I
       LET UA=RANDOM.F(NODE)
       LET ATIM(NODE)=(-1.0/LAMBDA(NODE))*(LOG.E.F(UA))
       SCHEDULE AN ARRV GIVEN NODE IN ATIM(NODE) UNITS
   LOOP
   SCHEDULE A RESET IN 10000.0 UNITS    ''TIME TO DELETE BIAS
   SCHEDULE AN OUTPUT IN 10150.0  UNITS  ''END OF FIRST BATCH
   START SIMULATION
STOP
END ''MAIN
EVENT RESET  ''DELETES BIAS, RESETS FOR NEXT BATCH
FOR EACH NODE RESET THE TOTALS OF ATIM, STIM, BUSY, WTIM
FOR I=1 TO 2, DO                  ''NO. OF NODES
    LET ATIM(I)=0
    LET STIM(I)=0
    LET WTIM(I)=0
    LET TCUST(I)=0
LOOP
FOR I=1 TO 2, DO
    FOR J=1 TO 2,DO
        LET RCUST(I,J)=0
        LET O.R(I,J)=0
    LOOP
LOOP
RETURN
END ''EVENT RESET
EVENT ARRV GIVEN NODE    ''EXTERNAL ARRIVAL AT GIVEN NODE
DEFINE NODE AS AN INTEGER VARIABLE
CREATE A CUST
LET NODT(CUST)= TIME.V
       LET UA=RANDOM.F(NODE)
       LET ATIM(NODE)=(-1.0/LAMBDA(NODE))*(LOG.E.F(UA))
       SCHEDULE AN ARRV GIVEN NODE IN ATIM(NODE) UNITS
```

```
IF BUSY(NODE)= S(NODE), FILE CUST IN QUEUE(NODE)
ELSE
    LET BUSY(NODE)= BUSY(NODE) + 1
    LET WTIM(NODE)=0
    LET AWQ(NODE)=0
    LET US=RANDOM.F(NODE+5)
    LET STIM(NODE)=(-1.0/MU(NODE))*(LOG.E.F(US))
    SCHEDULE AN EOS GIVEN CUST, NODE IN STIM(NODE) UNITS
ALWAYS
RETURN
END    ''EVENT ARRV
EVENT EOS GIVEN CUST, NODE    ''END OF SERVICE EVENT
DEFINE CUST, NODE AS INTEGER VARIABLES
LET TCUST(NODE)= TCUST(NODE) + 1
CALL ROUTE2 GIVEN CUST,NODE
RETURN
END   ''EVENT EOS
EVENT OUTPUT
DEFINE I, J AS INTEGER VARIABLES
FOR I=1 TO 2, DO
    FOR J=1 TO 2, DO
        LET O.R(I,J)= RCUST(I,J)/TCUST(I)
    LOOP
    WRITE 1.0/A.AR(I), 1.0/A.SR(I), O.R(I,1), O.R(I,2)
          AS 4 D(10,4) USING UNIT 1
    WRITE AS / USING UNIT 1
    WRITE A.BUSY(I)/S(I), A.WQ(I) AS 2 D(10,4)
          USING UNIT 1
    WRITE AS / USING UNIT 1
LOOP
LET I=1
LET C=C+1
PRINT 6 LINES WITH NM,C, I, I+1,1.0/A.AR(I), 1.0/A.AR(I+1),
1.0/A.SR(I),1.0/A.SR(I+1),A.BUSY(I)/S(I),A.BUSY(I+1)/S(I+1),
              A.WQ(I), A.WQ(I+1) THUS
RESULTS FOR MACRO **.*      BATCH **.*
NODE *                NODE *
ARRIVAL= **.****      ARRIVAL= **.****
SERVICE= **.****      SERVICE= **.****
    RHO=  *.****          RHO=  *.****
     WQ=***.****           WQ=***.****
PRINT 1 LINE THUS
OBSERVED ROUTING PROBABILITY MATRIX (O.R)
LIST O.R
SKIP 1 LINE
IF C LT 25.0
     SCHEDULE A RESET NOW
     SCHEDULE AN OUTPUT IN 150.0 UNITS
ELSE PRINT 1 LINE WITH G.AWQ(1), G.AWQ(2) THUS
OVERALL MEAN FOR WQ(1)=***.****        WQ(2)=***.****
SKIP 1 LINE
     LET C=0
     LET NM=NM+1
     FOR EACH NODE RESET THE TOTALS OF AWQ
     FOR I=1 TO 2, DO     ''NO. NODES
            LET AWQ(I)=0
     LOOP
     IF NM LE 10.0
          SCHEDULE A RESET NOW
          SCHEDULE AN OUTPUT IN 150.0 UNITS
```

```
        ELSE STOP
        ALWAYS
ALWAYS
RETURN
END     ''EVENT OUTPUT
ROUTINE ROUTE2 GIVEN CUST, NODE    ''FOR TWO NODE NETWORK
DEFINE CUST, NODE, NEXT AS INTEGER VARIABLES
LET DEST = RANDOM.F(5)
IF DEST LE R(NODE,1), LET NEXT=1
      LET NODT(CUST)=TIME.V
      LET RCUST(NODE,NEXT)= RCUST(NODE,NEXT) + 1
      IF BUSY(NEXT)= S(NEXT),   FILE CUST IN QUEUE(NEXT)
            ELSE    LET BUSY(NEXT)= BUSY(NEXT) +1
              LET WTIM(NEXT)=0
              LET AWQ(NEXT)=0
              LET US=RANDOM.F(NEXT+5)
              LET STIM(NEXT)=(-1.0/MU(NEXT))*(LOG.E.F(US))
              SCHEDULE AN EOS GIVEN CUST, NEXT IN STIM(NEXT) UNITS
      ALWAYS
ELSE  IF DEST LE R(NODE,2)
            LET NEXT = 2
            LET NODT(CUST) = TIME.V
            LET RCUST(NODE,NEXT)= RCUST(NODE,NEXT) + 1
            IF BUSY(NEXT) = S(NEXT)
                  FILE CUST IN QUEUE(NEXT)
            ELSE  LET BUSY(NEXT)= BUSY(NEXT) + 1
                  LET WTIM(NEXT)=0
                  LET AWQ(NEXT)=0
                  LET US=RANDOM.F(NEXT+5)
                  LET STIM(NEXT)=(-1.0/MU(NEXT))*(LOG.E.F(US))
                  SCHEDULE AN EOS GIVEN CUST, NEXT IN STIM(NEXT) UNITS
            ALWAYS
       ELSE  DESTROY CUST
       ALWAYS
ALWAYS
IF QUEUE(NODE) IS EMPTY
       LET BUSY(NODE)= BUSY(NODE) - 1
       RETURN
ELSE   REMOVE THE FIRST CUST FROM QUEUE(NODE)
       LET WTIM(NODE)= TIME.V - NODT(CUST)
       LET AWQ(NODE)= TIME.V-NODT(CUST)
       LET US=RANDOM.F(NODE+5)
       LET STIM(NODE)=(-1.0/MU(NODE))*(LOG.E.F(US))
       SCHEDULE AN EOS GIVEN CUST, NODE IN STIM(NODE) UNITS
RETURN
END  ''ROUTINE ROUTE2
ROUTINE SNAP.R
LIST TCUST
SKIP 1 LINE
LIST RCUST
SKIP 1 LINE
LIST ATTRIBUTES OF EACH EOS IN EV.S(I.EOS)
SKIP 1 LINE
LIST ATTRIBUTES OF EACH ARRV IN EV.S(I.ARRV)
RETURN
END ''SNAP.R
/*
```

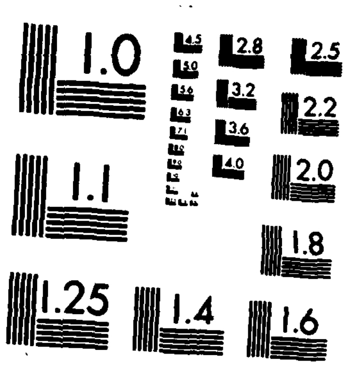
```
//     JOB  ,
//     REGION=768K
/*JOBPARM      LINES=5000
//S1    EXEC    FORTVCG,IMSLIB=SINGLE
//FORT.SYSIN     DD    *
CCC    PROGRAM ANALYZES SIMULATION RESULTS FOR VARIANCE REDUCTION
CCC    PROGRAM COMPUTES EFFECTIVE ARRIVAL RATES AND PERFORMANCE
CCC    MEASURES FOR A GIVEN NETWORK USING AN IMSL ROUTINE
CCC    TO SOLVE THE BALANCE EQUATIONS. CRUDE AND CONTROL
CCC    VARIATE ESTIMATES ARE COMPUTED AND THEIR VARIANCES
CCC    ARE COMPARED.
CCC    DEFINITIONS  OF MAJOR VARIABLES
CCC    NN              NO. OF NODES
CCC    NB              NO. OF BATCHES PER MACRO
CCC    NM              NO. OF MACROS PER EXPERIMENT
CCC    SUBSCRIPTS FOR DEFINITIONS
CCC    I=NO. OF NODES,  J=NO. OF BATCH,    K=NO. OF MACRO
CCC    RO(I)           LONG RUN UTILIZATION, NODE I
CCC    WQ(I)           LONG RUN QUEUE TIME, NODE I
CCC    NOTE: STATISTICS ARE DEFINED FOR RO ONLY; NOTATION IS
CCC          SIMILAR FOR  WQ
CCC    ROJ(I)          JACKSON STEADY STATE FOR RO(I)
CCC    ROA(I,J)         ANALYTIC CONTROL FOR RO(I,J)
CCC    ROS(I,J)        SIMULATION ESTIMATE FOR RO(I,J)
CCC    AROS(I,K)       MEAN FOR ROS(I,J) OVER MACRO K
CCC    AROA(I)         MEAN FOR ROA(I,J) OVER A MACRO
CCC    VROA(I)         VARIANCE FOR RO(I,J) OVER A MACRO
CCC    CRO(I)          COVARIANCE(ROS(I,J),ROA(I,J)) OVER A MACRO
CCC    BCRO(I)         ESTIMATED CONTROL COEFFICIENT, B, FOR RO(I)
CCC    BROC(I,K)       CONTROL ESTIMATOR FOR RO(I) IN MACRO K
CCC    ROC(I,K)        CONTOL ESTIMATOR,B=1, FOR RO(I) IN MACRO K
CCC    MROC(I)         MEAN OF ROC(I,K)
CCC    VROC(I)         VARIANCE OF ROC(I,K)
CCC    MBROC(I)        MEAN OF BROC(I,K)
CCC    VBROC(I)        VARIANCE OF BROC(I,K)
CCC    MAROS(I)        MEAN OF AROS(I,K)
CCC    VAROS(I)        VARIANCE OF AROS(I,K)
CCC    VRROC(I)        VARIANCE REDUCTION FROM ROC(I)
CCC    VRBROC(I)       VARIANCE REDUCTION FROM BROC(I)
CCC    MWQS(I,K)       OVERALL MEAN OF WQS AT NODE I, MACRO K
CCC    GMWQS(I)        OVERALL MEAN OF WQS AT NODE I FOR ALL K
CCC    NOTE: A LIST OF OTHER PROGRAM VARIABLES FOLLOWS
CCC    A(I,II)         EFFECTIVE ARRIVAL RATE MATRIX
CCC    B(I)            ARRIVAL(INPUT)/EFFECTIVE ARRIVAL(OUTPUT)
CCC    IA              ROW DIMENSION OF A(I,II)
CCC    IDGT            ACCURACY TO DECIMAL PLACE OF LEQT1F SOLUTION
CCC    IER             LEQT1F WARN FLAG ( ACCURACY OR SINGULARITY)
CCC    LEQT1F          IMSL LINEAR EQUATION SOLVER
CCC    M               NO. OF RIGHT HAND SIDES
CCC    N               NO. OF ROWS IN B(I)
CCC    R(I,II)         PR. OF ROUTING FROM NODE I TO II
CCC    S(I)            NO. OF SERVERS AT NODE I
CCC    WKAREA(I)       DIMENSION GT OR EQ TO N
CCC    MU(I)           SERVICE RATE AT NODE I
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCC    DECLARE AND DIMENSION VARIABLES
       INTEGER  NN, NM, NB, M, N, IER, IA, IDGT, S ,FACT,CRUDE
       REAL       MWQS(2,10),GMWQS(2),CVWQS(2)
       REAL    ROJ(2), ROA(2,50), AROS(2,10), AROA(2), VROA(2),
```

```
     +        CRO(2), BCRO(2), BROC(2,10), ROC(2,10), MROC(2),
     +        VROC(2), MBROC(2), VBROC(2), MAROS(2), VAROS(2),
     +        VRROC(2),   VRBROC(2)
      REAL    WQJ(2), WQA(2,50), AWQS(2,10), AWQA(2), VWQA(2),
     +        CWQ(2), BCWQ(2), BWQC(2,10), WQC(2,10), MWQC(2),
     +        VWQC(2), MBWQC(2), VBWQC(2), VRWQC(2), VRBWQC(2)
      REAL    B(2), WKAREA(4), MU(2), R(2,2), A(2,2)
CCC   LABELED COMMON STMT
      COMMON /SIMMEA/ ROS(2,25), WQS(2,25), S(2)
CCC   READ IN NETWORK PARAMETERS AND IMSL ARGUMENTS
      READ(5,*)NN, NM, NB, M, N, IA, IDCT
CCC   READ JACKSON PARAMETERS AND FORM A(I,II) MATRIX
      DO 20  I=1,NN
          READ(5,*)B(I), MU(I), S(I), R(I,1), R(I,2)
   20 CONTINUE
CCC   READ OVERALL MACRO MEANS FOR WQ
      DO  25  K=1,NM
           READ(5,*)MWQS(1,K), MWQS(2,K)
   25 CONTINUE
      DO 40  I=1,NN
         DO 30  II=1,NN
            IF(I.EQ.II)THEN
               A(I,II)=1-R(I,II)
            ELSE
               A(I,II)= -R(II,I)
            ENDIF
   30    CONTINUE
   40 CONTINUE
CCC   ECHO INPUT
      WRITE(6,41)
   41 FORMAT('0',15X, 'ECHO INPUT')
      WRITE(6,*)'NN=', NN, 'NB=', NB, 'NM=', NM, 'M=', M, 'N=', N,
     +          'IA=', IA, 'IDCT=', IDCT
      DO 60  I=1,NN
         WRITE(6,42)I, B(I), MU(I), S(I)
   42    FORMAT('0','NODE', I3,2X,'B(I)=',F10.4,2X,'MU(I)=',F10.4,
     +          2X,'S(I)=',I3)
         DO 50  II=1,NN
            WRITE(6,43)I, II, R(I,II), A(I,II)
   43       FORMAT('0','PROBABILITY FROM',I3,1X,'TO',I3,1X,'=',F10.4,
     +             1X,'A(I,II)=',F10.4)
   50    CONTINUE
   60 CONTINUE
CCC     SOLVE FOR JACKSON EFFECTIVE ARRIVAL RATES
CCC     CALL IMSL ROUTINE LEQT1F
      CALL LEQT1F(A, M, N, IA, B, IDCT, WKAREA, IER)
      IF(IER.GT.0)THEN
          WRITE(6,*)'ERROR FLAG=', IER, 'JACKSON'
          STOP
      ENDIF
CCC   COMPUTE JACKSON MEASURES
      J=1
      CALL PERF(NN, NB, J, B, MU, ROA, WQA, CRUDE)
      DO 70  I=1,NN
         ROJ(I)=ROA(I,J)
         WQJ(I)=WQA(I,J)
   70 CONTINUE
CCC   ECHO JACKSON MEASURES
      DO 110  I=1,NN
          WRITE(6,75)I, ROJ(I), WQJ(I)
```

```
 75     FORMAT('0','JACHSON VALUES NODE',I3,2X,'RO=',F10.4,2X,
     +          'WQ=',F10.4)
 110 CONTINUE
CCC   FOR EACH MACRO COMPUTE MEASURES AND CV'S
      DO 270  K=1,NM
CCC   FOR EACH BATCH READ READ PARAMETERS AND SIM. ESTIMATES
         DO 140  J=1,NB
CCC   FOR EACH NODE READ PARAMETERS AND ESTIMATES
            DO 130  I=1,NN
                  READ(3,*)B(I), MU(I), R(I,1), R(I,2)
                  READ(3,*)ROS(I,J), WQS(I,J)
 130        CONTINUE
            DO 136 I=1,NN
               DO 135  II=1,NN
                  IF(I.EQ.II)THEN
                     A(I,II)=1-R(I,II)
                  ELSE
                     A(I,II)= -R(II,I)
                  ENDIF
 135           CONTINUE
 136        CONTINUE
CCC   COMPUTE EFFECTIVE ARRIVAL RATE FOR A BATCH
CCC   CALL IMSL ROUTINE LEQT1F
            CALL LEQT1F(A, M, N, IA, B, IDGT, WKAREA, IER)
            IF(IER.GT.0)THEN
                  WRITE(6,*)'ERROR FLAG BATCH', J,K, '=', IER
                  STOP
            ENDIF
CCC     COMPUTE BATCH MEASURES FOR EACH NODE
            CALL PERF(NN, NB, J, B, MU, ROA, WQA, CRUDE)
 140     CONTINUE
          WRITE(6,141)K,CRUDE
 141      FORMAT('0','IN MACRO', I3,2X,'RHO GE 1.0',I3,2X,'TIMES')
CCC     COMPUTE MEANS, VAR'S, COV, BSTAR, FOR BATCH OUTPUT
            DO 240  I=1,NN
               DO 150  J=1,NB
                  AROS(I,K)= ROS(I,J)+AROS(I,K)
                  AWQS(I,K)= WQS(I,J)+AWQS(I,K)
                  AROA(I)=ROA(I,J)+AROA(I)
                  VROA(I)= ROA(I,J)*ROA(I,J)+VROA(I)
                  AWQA(I)= WQA(I,J)+AWQA(I)
                  VWQA(I)=WQA(I,J)*WQA(I,J)+VWQA(I)
 150           CONTINUE
CCC        COMPUTE MEAN, VAR FOR MEASURE AT NODE I
              AROS(I,K)= AROS(I,K)/(NB)
              VROA(I)=VROA(I)/(NB-1)-AROA(I)*AROA(I)/
     +              ((NB-1)*(NB))
              AROA(I)= AROA(I)/(NB)
              AWQS(I,K)= AWQS(I,K)/(NB)
              VWQA(I)= VWQA(I)/(NB-1)-AWQA(I)*AWQA(I)/
     +              ((NB-1)*(NB))
              AWQA(I)= AWQA(I)/(NB)
CCC     COMPUTE COVARIANCES
             DO 160  J=1,NB
                 CRO(I)= (ROS(I,J)-AROS(I,K))*(ROA(I,J)-AROA(I))+
     +                 CRO(I)
                 CWQ(I)= (WQS(I,J)-AWQS(I,K))*(WQA(I,J)-AWQA(I))+
     +                 CWQ(I)
 160         CONTINUE
CCC        COMPUTE COV AND BSTAR
```

AD-A171 280 THE EFFECTIVENESS OF JACKSON NETWORKS AS CONTROL VARIATES FOR QUEUEING NETWORK SIMULATION(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH A P SHARON 1986 2/2

UNCLASSIFIED AFIT/CI/NR-86-129T F/G 12/2 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
            CRO(I)=CRO(I)/(NB-1)
            CWQ(I)= CWQ(I)/(NB-1)
            BCRO(I)= CRO(I)/VROA(I)
            BCWQ(I)= CWQ(I)/VWQA(I)
CCC     COMPUTE CONTROL VARIATE ESTIMATES FOR RUN K
            BROC(I,K)= AROS(I,K)-BCRO(I)*(AROA(I)-ROJ(I))
            ROC(I,K)=AROS(I,K)-AROA(I)+ROJ(I)
            BWQC(I,K)= MWQS(I,K)-BCWQ(I)*(AWQA(I)-WQJ(I))
            WQC(I,K)= MWQS(I,K)-AWQA(I)+WQJ(I)
CCC       PRINT DATA SUMMARY FOR RUN K
            WRITE(6,170)I,K
  170       FORMAT('0', 'RESULTS FOR NODE', I3,5X,'MACRO', I3)
            WRITE(6,180)ROJ(I), AROS(I,K), CRO(I), BCRO(I),BROC(I,K),
     +                   ROC(I,K)
  180       FORMAT(' ', 'ROJ=', F10.4,2X,'AROS=', F10.4,2X,
     +      'COV(RO)=',F10.4,2X,'BSTAR=',F10.4,2X,'BROC=',F10.4,
     +      2X,'ROC=', F10.4)
            WRITE(6,200)WQJ(I), MWQS(I,K), CWQ(I), BCWQ(I),BWQC(I,K),
     +           WQC(I,K)
  200       FORMAT(' ', 'WQJ=',F10.4,2X,'AWQS=',F10.4,2X,'COV(WQ)=',
     +      F10.4,2X,'BSTAR=',F10.4,2X,'BWQC=',F10.4,2X,'WQC=',F10.4)
  240     CONTINUE
CCCC    INITIALIZE BATCH MEASURE ARRAYS
              DO 260  I=1,NN
               DO 250  J=1,NB
                   ROA(I,J)=0
                   WQA(I,J)=0
  250          CONTINUE
               AROA(I)=0
               VROA(I)=0
               CRO(I)=0
               BCRO(I)=0
               AWQA(I)=0
               VWQA(I)=0
               CWQ(I)=0
               BCWQ(I)=0
  260       CONTINUE
            CRUDE=0
  270  CONTINUE
CCC    COMPUTE VARIANCE REDUCTION SUMS
    DO 390  I=1,NN
         DO 280  K=1,NM
             MROC(I)= ROC(I,K)+MROC(I)
             VROC(I)= ROC(I,K)*ROC(I,K)+VROC(I)
             MWQC(I)= WQC(I,K)+MWQC(I)
             VWQC(I)= WQC(I,K)*WQC(I,K)+VWQC(I)
             MBROC(I)= BROC(I,K)+MBROC(I)
             VBROC(I)= BROC(I,K)*BROC(I,K)+VBROC(I)
             MBWQC(I)= BWQC(I,K)+ MBWQC(I)
             VBWQC(I)= BWQC(I,K)*BWQC(I,K)+ VBWQC(I)
             MAROS(I)= AROS(I,K)+ MAROS(I)
             VAROS(I)= AROS(I,K)*AROS(I,K)+ VAROS(I)
             GMWQS(I)= MWQS(I,K)+ GMWQS(I)
             GVWQS(I)= MWQS(I,K)*MWQS(I,K)+ GVWQS(I)
  280    CONTINUE
CCCCC    COMPUTE VARIANCE /MEANS
           VROC(I)= VROC(I)/(NM-1)-MROC(I)*MROC(I)/((NM-1)*NM)
           MROC(I)= MROC(I)/NM
           VWQC(I)= VWQC(I)/(NM-1)-MWQC(I)*MWQC(I)/((NM-1)*NM)
           MWQC(I)= MWQC(I)/NM
```

```
               VBROC(I)=VBROC(I)/(NM-1)-MBROC(I)*MBROC(I)/((NM-1)*NM)
               MBROC(I)= MBROC(I)/NM
               VBWQC(I)=VBWQC(I)/(NM-1)-MBWQC(I)*MBWQC(I)/((NM-1)*NM)
               MBWQC(I)= MBWQC(I)/NM
               VAROS(I)=VAROS(I)/(NM-1)-MAROS(I)*MAROS(I)/((NM-1)*NM)
               MAROS(I)= MAROS(I)/NM
               GVWQS(I)=GVWQS(I)/(NM-1)-GMWQS(I)*GMWQS(I)/((NM-1)*NM)
               GMWQS(I)= GMWQS(I)/NM
CCC     COMPUTE VARIANCE REDUCTION
              VRROC(I)=(VAROS(I)-VROC(I))/VAROS(I)
              VRBROC(I)=(VAROS(I)-VBROC(I))/VAROS(I)
              VRWQC(I)=(GVWQS(I)-VWQC(I))/GVWQS(I)
              VRBWQC(I)=(GVWQS(I)-VBWQC(I))/GVWQS(I)
CCC     PRINT VARIANCE RDUCTIONS
          WRITE(6,290)I
  290     FORMAT('0', 'VARIANCE REDUCTION AT NODE  ', I3)
          WRITE(6,300)ROJ(I), MAROS(I), VAROS(I)
  300     FORMAT(' ', 'ROJ=', F10.4,3X, 'MEAN SIM RO=', F10.4,3X,
     +             'VAR(SIM RO)=', F10.4)
          WRITE(6,310)MBROC(I), VBROC(I), MROC(I),VROC(I)
  310     FORMAT(' ', 'MEAN BROC=', F10.4,3X, 'VAR(BROC)=', F10.4,
     +             3X, 'MEAN ROC=', F10.4,3X, 'VAR(ROC)=', F10.4)
          WRITE(6,320)VRBROC(I), VRROC(I)
  320     FORMAT('0', 'VAR REDUCE BROC=', F10.4,5X,'VAR REDUCE ROC=',
     +             F10.4)
          WRITE(6,330)WQJ(I), GMWQS(I), GVWQS(I)
  330     FORMAT('0', 'WQJ=', F10.4,3X,'MEAN SIM WQ=', F10.4,3X,
     +             'VAR(SIM WQ)=', F10.4)
          WRITE(6,340)MBWQC(I), VBWQC(I), MWQC(I), VWQC(I)
  340     FORMAT(' ', 'MEAN BWQC=', F10.4,3X,'VAR(BWQC)=', F10.4,
     +             3X,'MEAN WQC=', F10.4,3X,'VAR(WQC)=', F10.4)
          WRITE(6,350)VRBWQC(I), VRWQC(I)
  350     FORMAT(' ', 'VAR REDUCE BWQC=', F10.4,5X,'VAR REDUCEWQC=',
     +             F10.4)
  390 CONTINUE
      STOP
      END
CCC     INTEGER FUNCTION TO COMPUTE A FACTORIAL
      INTEGER FUNCTION FACT(ISERV)
        KSERV=1
        IF(ISERV.GT.1)THEN
           DO 500  I=1,ISERV
              KSERV=KSERV*I
  500      CONTINUE
        ENDIF
        FACT=KSERV
        RETURN
        END
CCC   SUBROUTINE TO COMPUTE PERFORMANCE MEASURES
      SUBROUTINE PERF(NN, NB, J, B, MU, ROA, WQA,CRUDE)
        COMMON  /SIMMEA/ROS(2,25), WQS(2,25), S(2)
        INTEGER NN,NB,J, FACT, S, CRUDE
        REAL  B(2), MU(2), ROA(2,25), WQA(2,25), LQA
CCC     COMPUTE MEASURES FOR EACH NODE
        DO 820  I=1,NN
           T1=0
           ROA(I,J)=B(I)/(S(I)*MU(I))
CCC   IF RO GREATER THAN ONE, REPLACE RO WITH .9999
CCC     TO COMPUTE MEASURES
             IF(ROA(I,J).GE.1)THEN
```

```
                    ROA(I,J)=.9999
                    CRUDE= CRUDE+1
                 ENDIF
                 T2=((ROA(I,J)*S(I))**S(I))/(FACT(S(I))*(1-ROA(I,J)))
                 DO 810  IJ=0,(S(I)-1)
                    T1=((ROA(I,J)*S(I))**IJ)/FACT(IJ) +T1
 810             CONTINUE
              PR0= 1/(T1+T2)
              LQA=PR0*((ROA(I,J)*S(I))**S(I))*ROA(I,J)/
     +              (FACT(S(I))*((1-ROA(I,J))**2))
              WQA(I,J)= LQA/B(I)
 820       CONTINUE
           RETURN
           END
//SYSIN    DD  *
//GO.FT03F001  DD DSN=TS3935.T2HU,DISP=SHR
```

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END

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